The Mechanics of Elastic Continuous Rotors with Reference to Balancing of Large Turbine and Generator Rotor Shafts with Mass Eccentricities or Bows

”Development of Practical Balancing Method of Flexible "Continuous" Rotors on “High Speed Balancing Machines” in Three Planes| Using "QHSB Method"

Study Draft

by:

Zlatan Racic

July 26th , 2021
updated:

June 12th, 2024

TABLE OF CONTENTS

I. Preface

System "1st critical speed " and "Quantum field theory"

Rotors balancing on balancing machines was always more trade than science. Nevertheless, balancing rules were derived from theory of rotordynamics, to facilitate balancing processes and establish the acceptable vibrations criteria in industry.

Rotors as part of the system of rotating machine is energy converter, transferring external torque force to rotor's mass to circular motion and to useful work, returning back to external environment. At same time, rotating mass static and dynamic equilibrium when in state of rest is disturbed from its natural symmetry in quantum field. that causes a possible interactions (in quantum field theory) that are governed by a few basic principles: locality, symmetry and renormalization group flow (the decoupling of short distance phenomena from physics at larger scales). The disturbance and renormalization is best observed as rotor is accellerated throgh the system 1st critical velocity range (forced response from "continuous" solid body mass rotor-shaft, horizontally oriented, constrained by gravity in oil bearings on elastic supports).

Rotor-shaft forced response eigenvector, which resembles half wave of rotor-shaft fundamental harmonic resonance frequency in free state, is caused by external torque force, when inherent eccentricitiy exist between non-rotating(inertial), and rotating (non-inertial) relative reference frames in quantum field as eccentricity force vector (e) advances in time (lead convention) ahead of pseudo linear deflection of flexible rotor-shaft, observed externally in Newton (global) coordinates as relative phase shift at ~ 90 degrees which serves as reference link between maximum peak response (deflection) in quantum field, and as observed in global coordinates. By rules of orthogonality, deflection lags velocity, and acceleration leads velocity. By Newton law of motion motion vector follows direction of force, balancing of such rotor-shaft should be done by force 180 opposite the motion (deflection). Since excitation force(s) are active along length of rotor between gravity contraints (eccentric mass axis as non-inertial reference frame), balancing correction masses should be placed axially distributed in three planes forming a virtual, dynamic mass axis forming miroring the RMS mass axis of innate eccentricity relative to rotor-shaf t rotational CL axis.

This is the basic logic for developing a QHSBM (Quasi-High Speed Balancing Method), for balancing flexible turbine and generator rotors operating in oil bearings on elastic supports, and based on displacement indication observed in global coordinates. As rotor-shaft is accelerated within several revolutions reference frames revert and renormalization proceeds continues until phase shift reaches 180 degrees and rotor reaches the state of "self-balancing " and self centering in "space and time" (within bearings and seals clearance). When correction masses magnitude are axially proportionally distributed to reflect the rotor-shaft axial asymmetries, rotor should be balanced for any critical speed and for operating speed.

The effects of force vectors and moments in quantum field are presented in figures A and B.

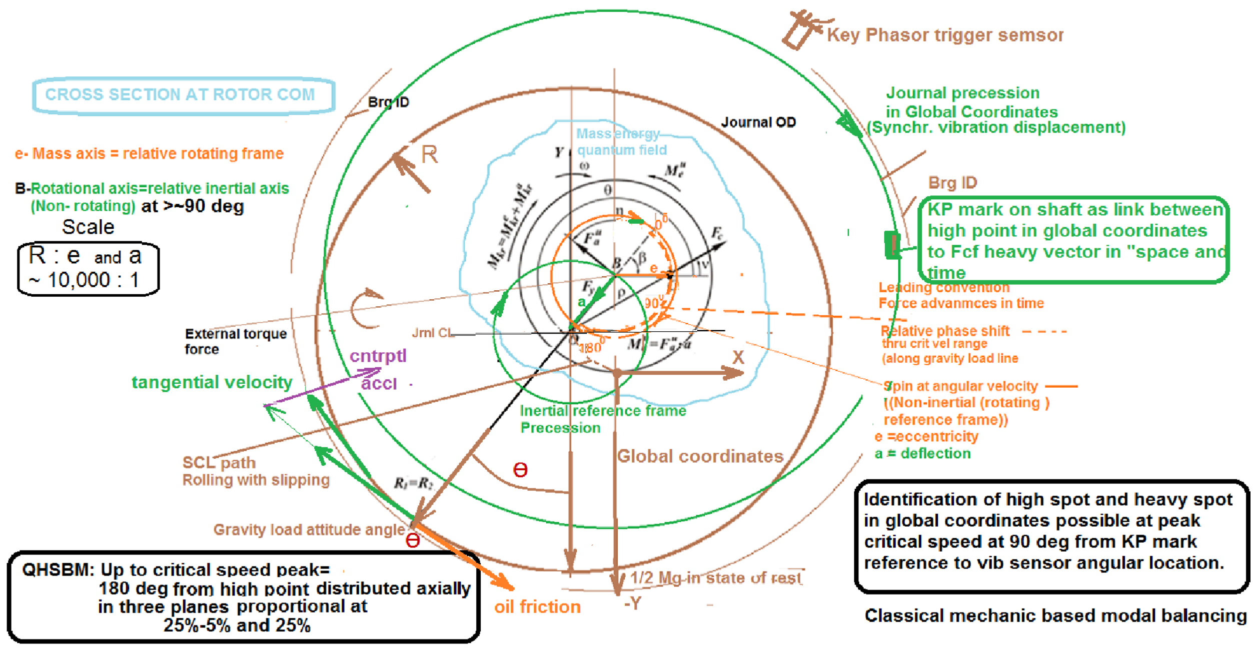


Figure A

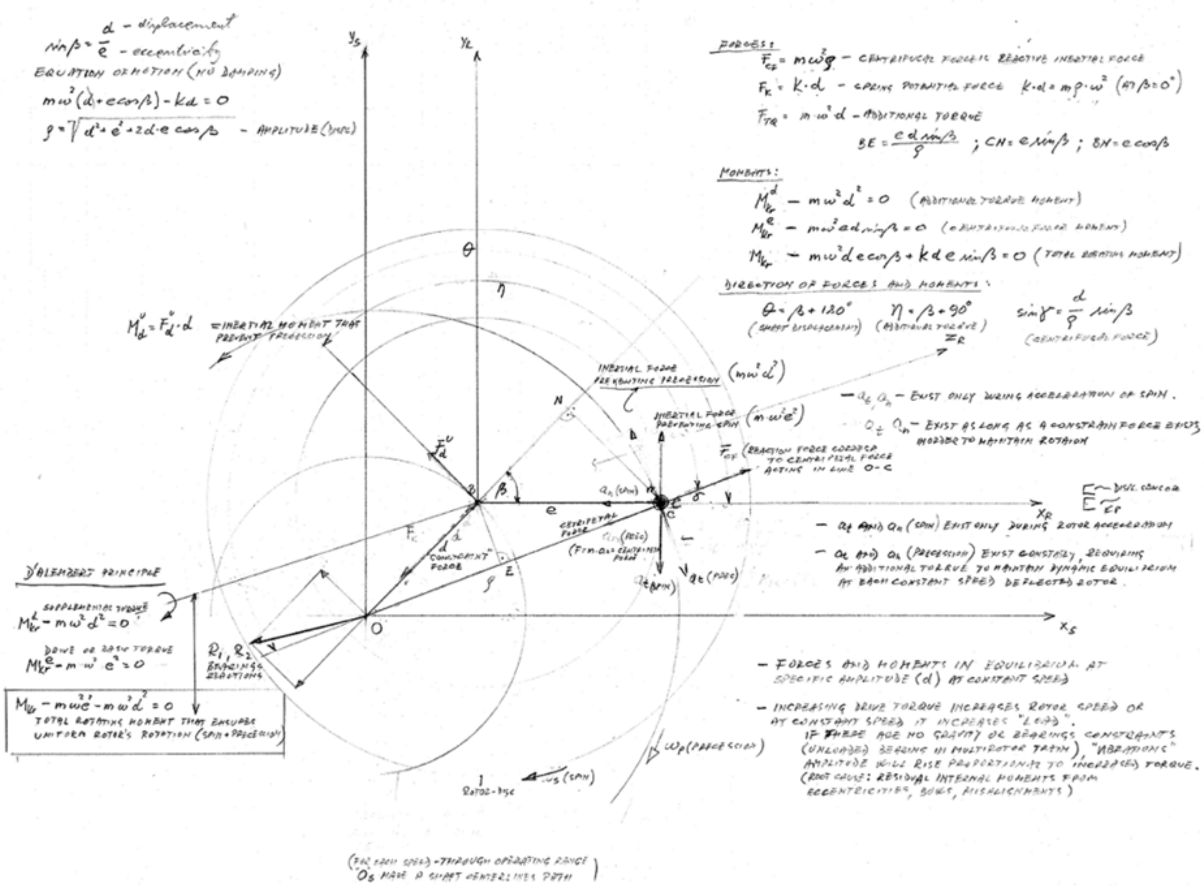


Figure B

II. Abstract
Rotordynamics and Balancing

Although there is enough material printed over decades of textbooks, journals, papers and articles on design and kinematics of rotating machinery predicting their dynamic behavior based on theories of Classical Mechanics as a basis in studying from Dynamics textbooks, there is practically no references in Dynamics text books in study of **kinetics of continuous rotors with significant runouts, measurable with dial indicator, as a systems in technological processes methods in practice, as in balancing of large turbine and generator rotors.** Turbine and generator rotors as units for power generation in power plants are designed through decades, ever larger through use of longer, thinner, and more efficient machines with flexible shafts made of new materials, and operating at super critical velocities. **The shaft (rotor) process in manufacturing is optimal if the shaft balance procedure can be conducted at subcritical speeds as a flexible, pseudo statically deflected "rigid" body on elastic supports [18]**, rather than current industry standard balancing methods requiring a compromise of a more costly and complicated high speed balancing process on balancing machines, first at system 1st critical speed, and then separately at supercritical speeds, under generally accepted **assumption by theoreticians, that rotor body displacements amplitudes are rotor's body harmonic modal linear oscillations, excited by a fictitious oscillating linear radial force, orthogonal to rotor mass axis referenced to Newton absolute coordinates.** The intent of this article is to introduce a new way of understanding of interaction, or cross-coupling of energy transports in rotating machines as in an open system, between external forces (torque), referenced to inertial, or non-rotating reference frame, and rotating , non-inertial centroidal RMS rotor's body mass axis reference frame, both referenced to absolute, fixed, Newtonian global coordinates, relative to which a pseudo-static shaft centerline path is a reference center in "space and time" of measured dynamic lateral orbital precession motion of RMS mass centerline axis, at subcritical angular velocities[14]. Such view have lead the author to develop a systematic technological process for "balancing" turbine rotors at subcritical velocities, up to ~ 90 degrees of peak displacement, before the inertial reference frames, and rotor switches from gravity constrained to Newton's natural rotating motion. [3, 4]. Such balancing procedure would permit a high speed shaft system to transition simultaneously through the system 1st and 2nd critical speeds, and through additional **system** critical speeds, and to operate in static and dynamic equilibrium, safely and stable, up to design operating speed and load.

Key words: ***Fundamental Energies, Force, Mass, Acceleration, Frequency, Fundamental Harmonic Resonance, Critical Speeds, Inertial reference frames, Gravity constraints, Balancing***

III. Introduction

Fundamental Principles of the Research on Balancing Continuous Large Turbine and Generator Rotors

Classical rotordynamics, also known as **rotor dynamics**, is a specialized branch of applied mechanics concerned with predicting the behavior of rotating structures during design process, and diagnosis of rotating machines prototypes vibration responses in operation. It is commonly used to analyze the behavior of all types rotating machines from jet engines and steam turbines and generators, high speed compressors to super high speeds turbo chargers. At its most basic level, rotor dynamics is concerned with one or more mechanical structures (rotors) supported by bearings, influenced by internal rotor mass that rotate assumed around a single rotational axis (no centroidal mass eccentricity, relative to center of rotation. The supporting structure is called a stator consisting of bearings and bearings supports. As the speed of rotation increases from standstill, the observed amplitude of vibration of rotor displacement , and forces transmitted to structure, often passes through a maximum that is called a critical speed. In theory the amplitude is assumed to be excited by "unbalance" of the rotating structure (rotor) at particular harmonic mode. If the amplitude of vibration at these critical speeds is excessive, then catastrophic machine or structure failure occurs. In addition to this, turbo machinery often develop instabilities which are related to the internal makeup of turbo machinery, and which must be corrected by tuning support dynamic stiffnesses relative to rotor stiffness. This is the chief concern of engineers who design large rotors.
<u>The real dynamics of the machine is difficult to model theoretically</u>. The calculations are based on simplified models which resemble various structural components (lumped parameters models), equations obtained from solving models numerically (Rayleigh–Ritz method) and finally from the finite element method (FEM), which is another approach for modeling and analysis of the machine for critical speeds of the rotor. There are also some analytical methods, such as the distributed transfer function method, which can generate analytical and closed-form natural frequencies, critical speeds and unbalanced mass response. On any machine prototype it is tested to confirm the precise frequencies of system critical speeds response, and then redesigned and tuned bearings and supports stiffness to assure that rotor fundamental harmonic resonance does not occur at system 1st critical speed.
Balancing methods derived from theoretical rotordynamics rules are being utilized by manufacturers of their particular rotating machines. These methods are based on the assumption that rotor mass rotate around a single axis relative to global fixed coordinates. These assumptions are valid for most rotating machines with "rigid" rotors and with flexible rotors operating at high speeds, with mass (inertia forces) that are negligible relative to machine power density. The balancing methods accepted as industry standards are valid also for low speed and low power density over rotors with large mass (inertia), like turbine and generator rotors in manufacturing environment, where rotors are machined to design tolerances.

The experience from service field has shown over many years that balancing of service rotors with bows, damaged couplings and in general the rotors which mass centroidal axis deviates from design tolerances and exceeds permissible eccentricity at 1st critical speed, of that as specified in ISO 1940-1 standard, balancing of such rotors always requires a compromise and trade off between balance condition achieved at 1st critical speed and at operating speed. A new balancing method QHSBM in 2N+1 planes, developed by author, and confirmed by experiments, brings rotor balanced condition on balancing machine to vanish forces and displacements of journals at any speed.

Large turbine and generator rotors with large moment of inertia are open systems [25], connected via rigid couplings as the part of rigid or flexible shaft overhang into an operating TG rotors train. Rotors are energy converters that convert external energy (torque) to useful work, e.g. generating electricity, at the specific angular velocity and efficiency of conversion [1]. Work is being transmitted via solid body continuous rotor mass (the sum of micro masses in quantum field, bound radially in lattices, binding lattices axially by electromagnetic energy with natural tendency to maintain particle masses in state of dynamic and static equilibrium [11]. Based on rotor's material atomic properties dynamic stiffness, and geometric body ratio of L/D, horizontally oriented between elastic supports as gravity constraints, rotors can be distinguished as "rigid" or "flexible", when operating in oil bearings, supported on elastic supports [5,6,7,8,9]. Balancing of such rotors individually on balancing machines should differ from the established methods used, developed based on classical mechanics and rotordynamics theory of two dimensional, linear oscillating motion of "mass, spring, and oil damping" model. The following statements reinforce the idea for need to develop a new balancing method, which at same time also satisfies the laws of nature, Newton laws, and conservation laws of energy, momentum and angular momentum.

1. ......*"If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.* ***Nikola Tesla***

<u>2. Open and closed systems and Ludwig von Bertanllaffy "Systems Theorem" [25]:</u>

"*Von Bertanllaffy noted that all systems in nature are ubiquitously open systems while <u>all systems studied by physicists are closed</u> as they do not interact with the outside world . When a physicist makes a model of the solar system, of an atom, or of a pendulum, or a rotor/bearing system, he or she assumes that all masses, particles, forces that affect the system are included in the model. It is as if the rest of the universe does not exist. This makes it possible to calculate the future states of the model with almost perfect accuracy, as used in machine design, (and theoretically predicting rotor dynamic behavior since all necessary information is known). However, such an assumption is only valid in practice when all assumptions that were considered in a closed system with boundary conditions, are fulfilled in practice. <u>When these assumptions are not fulfilled in practice that brings a discord between theory and practice. [op. ed.: in practice problems are solved within economic compromise]."</u>*

<u>3. Noether's Theorem of Systems Symmetry [26]:</u>

***Noether's theorem states that to the every continuous symmetry of a physical theory there corresponds a conserved quantity, i.e. a physical quantity that does not change with time. This is a tremendously important result, since it allows us to derive conserved quantities from the mathematical form of our theories* [2].**

A. Mechanics of Continuous Rotors

Difference Between Kinematics and Kinetics

Kinematics vs. Kinetics

Kinematics and Kinetics are two words in the study of motion and forces that are involved in these motions that confuse a lot of people. The situation becomes confusing because these two words are similar sounding, and also because both of them are involved in the study of motion. However, while kinematics is solely focused on the study of motion and does not take into account any forces that may be acting upon the body in motion, kinetics study the motion and the forces that are underlying this motion.

Kinematics

A study of motion in terms of kinematics, makes heavy use of the laws of motion such as Newton's first law which states that an object in a state of motion remains in motion unless and until an external force is applied to stop it. Kinematics is central in Classical Mechanics, in design and prediction of dynamic mechanical behavior of rotating bodies (rotors) modeled as a point mass in a two dimensional closed system, rather than as a continuous bodies. The equation of motion of the point mass in a closed system considers motion as a "state".

Kinetics

Kinetics comes from the Greek word kinesis which means pertaining to movement, and it is the study of motion and its causes. It is a branch of mechanics that deals with the analysis of kinetics and diagnostics of the mechanical behavior of rotating bodies (rotors) modeled as a continuous body mass rather than as point mass. The motion of the mass is considered as a "process".

.

Stress and Strain

In continuous rotors at REST in gravity environment, **stress** is a physical quantity that expresses body inherent energy in quantum field, and the internal forces that neighboring particles of a continuous material exert on each other resisting the effect of gravity acceleration. **Strain** is the measure of the deformation of the body material from action of external forces.

The relation between mechanical stress, deformation, and the rate of change of deformation is quite complicated. A linear approximation is used in Classical mechanics since it may be adequate in practice if the quantities are small enough. Stress that exceeds certain strength limits of the material will result in permanent deformation such as plastic bow or fracture, or even its crystal structure changing the surface hardness of rotors.

B. Viewing Rotors as 3D Continuous Rotating Solid Body
(Standing waves in Timoshenko Beam Model on Elastic Supports)

Balancing methods developed and utilized in industry over more than fifty years (parallel with introduction of more and more flexible rotors and more efficient rotating machines), based on classical mechanics theory for balancing horizontally oriented "unbalanced" rotors as a **closed system**, are based on classical mechanics theory, which theory's primary purpose is intended as a tool in rotating **machinery design predicting rotor dynamic behavior**.

Flexible rotors in state of rest assume gravity sag (elastic static deformation [18] as the 1st mode eigenvector, with radial deformation and axial strain proportional to rotor material dynamic axial stiffness perpendicular to reactive forces of gravity at supports $F= \sum m*g$. The intent of "balancing" accelerated rotor with eccentric centroidal mass axis, is to bring "radial mass symmetry" in quantum field, to minimize the observed journals precession orbit of geometric body (observed relative to global coordinates), by compensating horizontal radial elastic static deflection RMS axis, axially in three planes simultaneously, when rotor is accelerated at subcritical velocities through the **system's 1st critical velocity range.**

The current balancing methods used as industry standards, are commonly known as a "Static" and "Couple" balancing of "Euler beam model" at specific "critical speeds", and as Influence Coefficients method of point balancing, at higher, specific speed, mostly used in field balancing. When rotor is accelerated from state of rest, rotor motion is assumed in classical mechanics as 2D linear harmonic oscillation of journal as a point mass in direction of applied force, and with mathematically developed equation of motion, a motion is calculated with assumed system parameters "m, k, d" at frequency of "external fictitious force". At the peak of measured displacement amplitude of the system 1st critical speed, it is assumed that a "mass point unbalance" rotating CF vector is leading rotor deflection by 90 degrees. "Balancing" by "standard methods of the of rotor at 1st critical speed "modal" eigenvector is then performed by placing a counter mass additional 90 degrees against rotation from the antinode(s) of particular mode at particular angular rotor velocity referenced to Newtonian global coordinates. This is valid approach but only for solving rotor response at system 1st critical speed. What is not recognized in theory of rotordynamic based on Classical mechanics, is that **precessing centroidal mass axis** of rotor with axially distributed eccentricities, becomes a non-rotating reference frame after peak amplitude and `90 degrees phase, following the Newton 1st law of Motion.

When rotor is accelerated above 1st critical velocity to supercritical velocities, centroidal mass axis self-centers about rotor COM, following the natural law of axially randomly distributed mass symmetries.

After years of research and practical application of new author's developed "QHSBM" balancing method of 1st critical speed responses in three planes, which requires understanding definitions like: what excites phonons resonance in quantum mechanics, what creates the longitudinal wave and resonance of musical instrument string in physics of sound, what excites a longitudinal semi infinite wave velocity of Timoshenko beam model of length L in state of rest. The test method can be applied to determine fundamental harmonic resonance of continuous rotor in free state. It is necessary to know rotor natural harmonic resonance frequency, in order to design rotating machine system and rotor operating in oil bearings, on elastic supports, by modeling rotor as Timoshenko beam model (TBM), to operate the machine at angular velocity of the system 1st critical speed at less than 50% below the frequency of rotor's fundamental harmonic resonance frequency to operate stably.

In order to better understand the root cause of the systems forced frequency linear response in gravity environment at the system 1st critical speed, the following postulates were written by author justifying the need to develop a new balancing method of turbine and generator rotors at subcritical velocities.

B.1. What is "Fundamental Harmonic Resonance Frequency" Linear oscillating Motion Response

A continuous cylindrical solid body in free state of rest, in gravity environment, (e.g. rotor or guitar string), rigidly constrained at ends of physical length L, fundamental harmonic resonance frequency is internal inherent property of that body which can be excited by external kinetic energy added to body (mass) potential energy. Fundamental harmonic resonance frequency e.g. of guitar string is proportional to guitar string tension (lb / in^2) between nodes, geometric ratio of L/D and radial material atomic dynamic density . Continuous rotor fundamental harmonic resonance can be determined by identical approach. The main difference between e.g. guitar and turbine rotor is in their L/D ratio (~ 500 for string and ~8 for rotor shaft), and the ratio of internal tension over total body mass T/M.

B.1. a. Musical instrument string (L/D ~ 500)

Musical instrument string (guitar string), as any continuous solid body has inherent harmonic resonance, which can be excited when it is rigidly constrained within distance L between constraints. Resonance frequency can be tuned by applying tension of "body" between two constraints, and applying external impulse striking string by finger or pick, perpendicular to string length L (kinetic energy added to system potential energy. The harmonic frequency is unique to every solid body and it depends on material atomic structure of that body and physical dimension ratio L/D as shown in Figure 1. and 2.

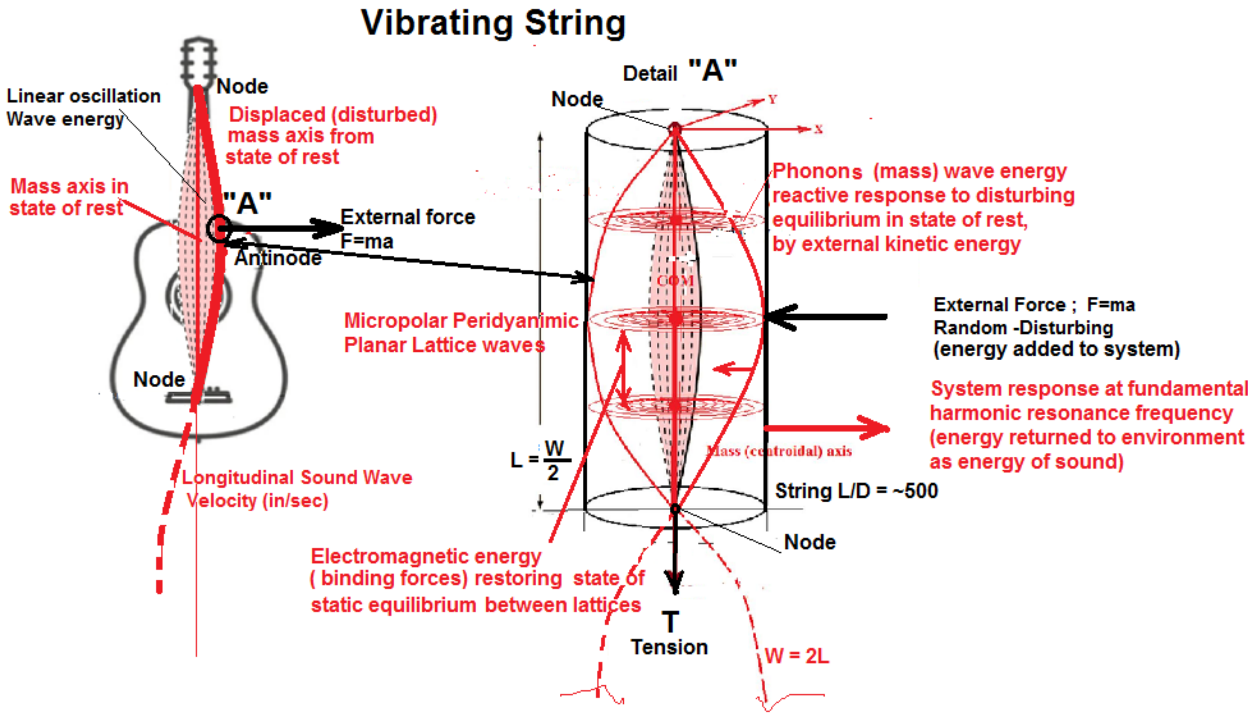


Figure 1. Resonance as result of energy conversion between systems

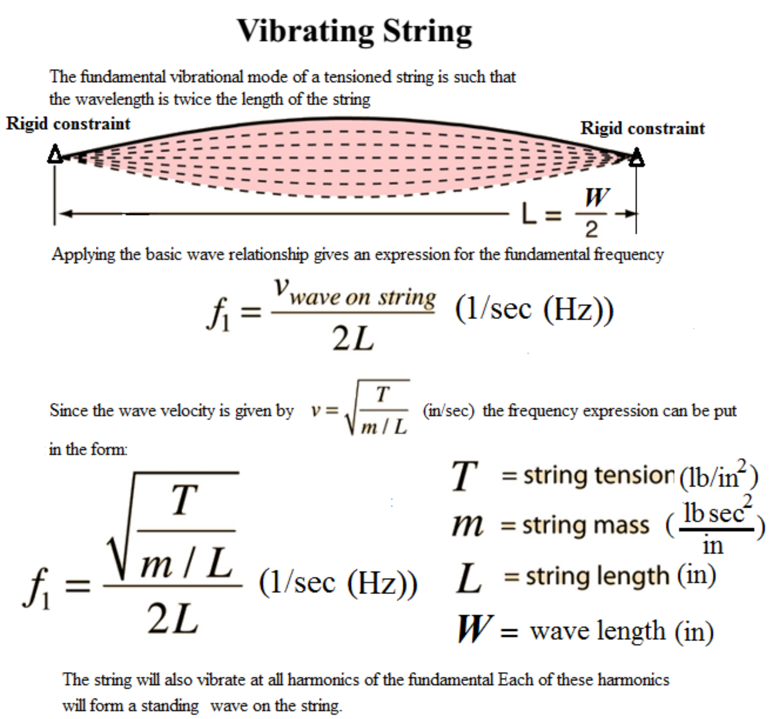


Figure 2. Calculation of string fundamental harmonic frequency(L/D~ 500)

B.1.b. Fundamental Harmonic Resonance Frequency of Continuous Rotating Solid Body in State of Rest(L/D ~ 7-9)

A major difference between musical instrument string and turbine rotor, in determination of the unique, inherent continuous solid body fundamental harmonic resonance frequency is in L/D ratio and body material total mass. A guitar string, even when coiled in packet still has inherent natural FHRF (Fundamental Harmonic Resonance Frequency), but the resonance can be excited only after it is rigidly constrained with potential energy at certain distance L, and by adding kinetic energy to "system" by external impact force. A turbine rotor vertically oriented in gravity environment and in state of "absolute" rest relative to earth, is rigidly constrained internally at its ends (nodes) between length L, having potential energy P :

$$P = W/g$$

When rotor is impacted by external force K (without physical disturbance of body from state of rest), the total energy in rotor E is the sum of :

$$E = P + K$$

Rotor as energy converter within an open system, converts added kinetic energy into sound energy, which is returned to external environment. E.g. impact test of rotor.

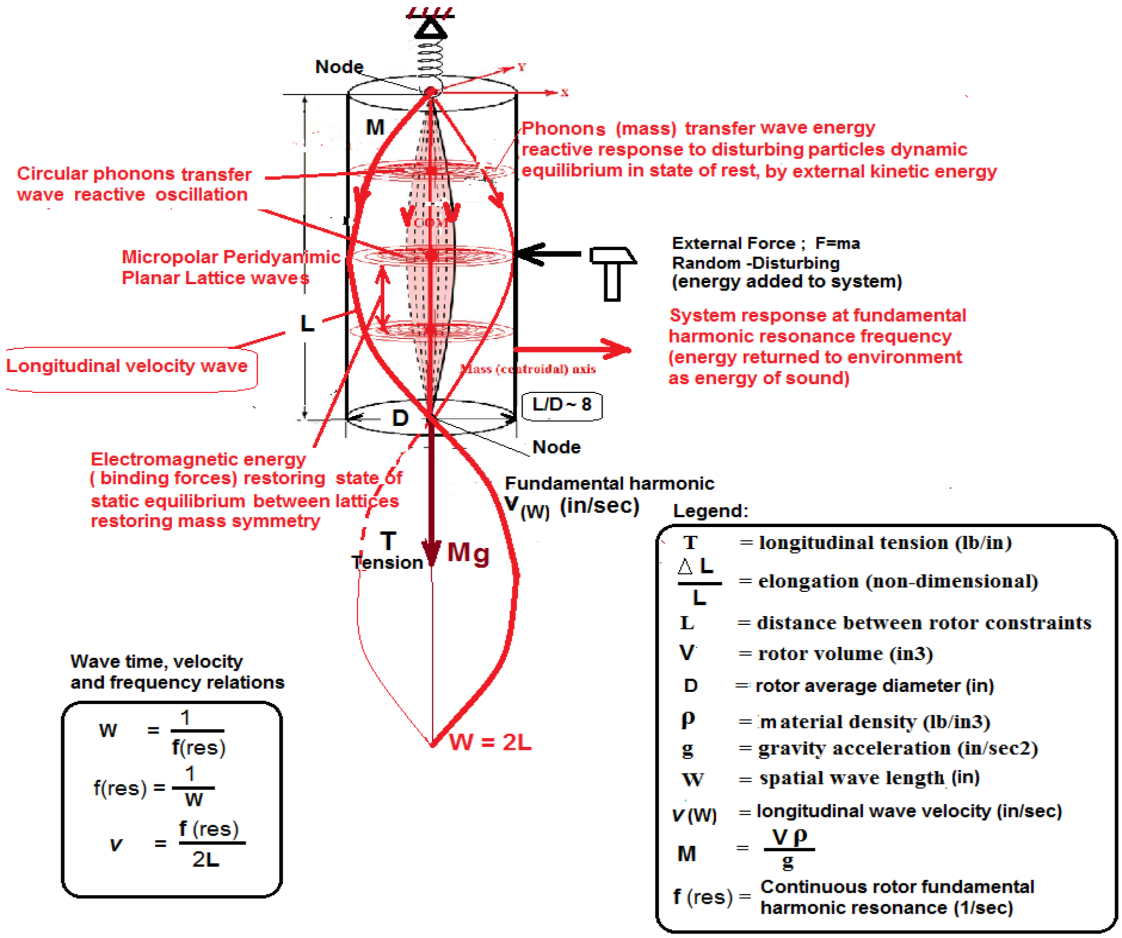
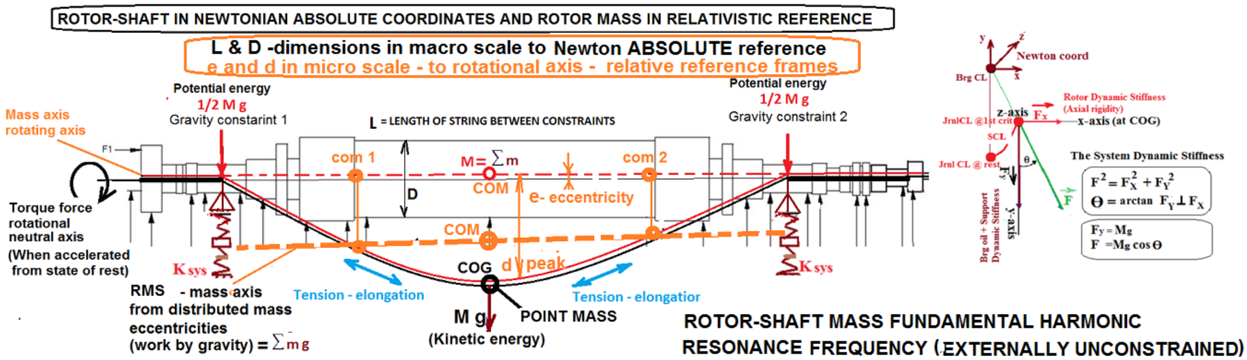


Figure 3 **Fundamental harmonic resonance frequency of a continuous rotor in free state (L/D~8)**



ROTOR-SHAFT MASS FUNDAMENTAL HARMONIC RESONANCE FREQUENCY (EXTERNALLY UNCONSTRAINED)

Since the wave velocity is given by $v = \sqrt{\dfrac{T}{m/L}}$, the frequency expression can be put in the form:

$$f_1 = \frac{\sqrt{\dfrac{T}{m/L}}}{2L}$$

T = string tension

m = string mass

L = string length

The string will also vibrate at all harmonics of the fundamental. Each of these harmonics will form a standing wave on the string.

SYSTEM CRITICAL SPEED

SOLID MASS CONTINUOUS FINITE LENGTH FLEXIBLE ROTOR SHAFT CONSTRAINED BY GRAVITY ON ELASTIC SUPPORT (TIMOSHENKO BEAM MODEL - SYSTEM CRITICAL SPEED

ω angular velocity; ω Linear brg oilstiffness + Linear support stiffness in direction of the apparent gravity

$$f_{(1ST\ CRIT)} \propto \frac{\sqrt{\dfrac{c\left(\frac{\tau}{\gamma}\right) \Big/ M\left(\frac{V\rho}{g}\right)}{L}}}{2L} \;(+\perp)\; \sqrt{\frac{K_{brg}}{M_{brg}}} + \sqrt{\frac{K_{sup}}{M_{sup}}} \;\left(\frac{rad}{sec}\right)$$

In State of Rest
Gravity Deflection of COM

Inherent rotor resonance- in state of rest

In Frequency Domain

(where: $m_{rot\text{-}int} = V\rho/g$)

$$\omega_{res\,(n)} \propto \sqrt{\frac{E}{m\,\rho}} = Hz\left(\frac{1}{sec}\right)$$

(Euler beam model- radial shear)

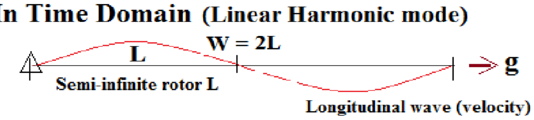
Legend:

- T = longitudinal tension (lb/in2)
- △L = elongation (non-dimensional)
- V = rotor volume (in3)
- ρ = Material density (lb/in3)
- g = gravity acceleration (in/sec2)
- W = frequency spatial wave length)
- L = distance between rotor constraints

Impulse and momentum On rotor In Free State (Internal Constraints)

Natural fundamental harmonic resonance frequency vawe of continuous solid body suspended at pivot and with mass axis in direction of gravity, excited by linear momentum perpendicular to mass axis

In Time Domain (Linear Harmonic mode)



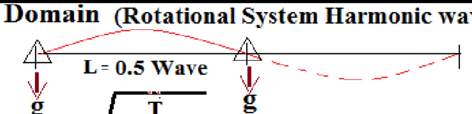
$$\omega_{res\,(n)} \propto \sqrt{\frac{\dfrac{T}{\dfrac{V\rho}{g}}}{L}} = \frac{in}{sec}\ \text{(velocity of sound)}$$

(Timoshenko beam model- longitudinal elongation — strain)

Impulse and Momentum On Horizontal Externally Constrained Rotor by Gravity

Fundamental harmonic resonance modal vawe of continuous rotor consisiting of two modal elements, constrained at nodal points, excited by steady force in rotating frame perpendicular to mass axis @ ω (rot)

In Time Domain (Rotational System Harmonic wave)



$$\omega_{res(rot)} \propto \frac{\sqrt{\dfrac{\dfrac{T}{\dfrac{V\rho}{g}}}{L}}}{2L} = \frac{Rad}{sec} \;@\; \frac{\omega\,(rot)}{\omega_{res\,(n)}} = 0.5$$

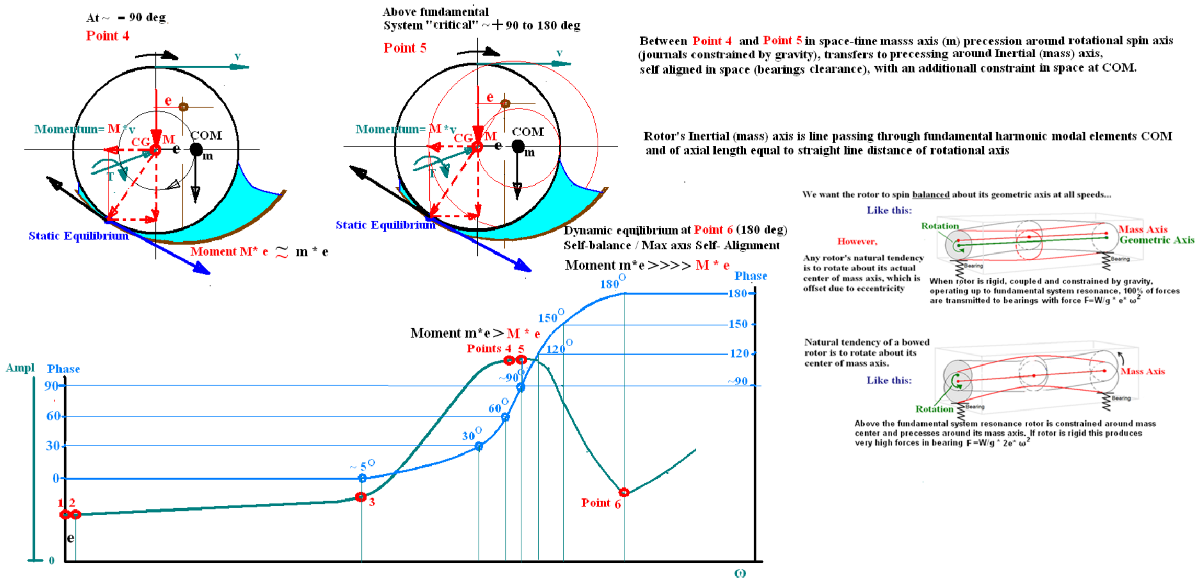
(Timoshenko beam model- radial resonance)

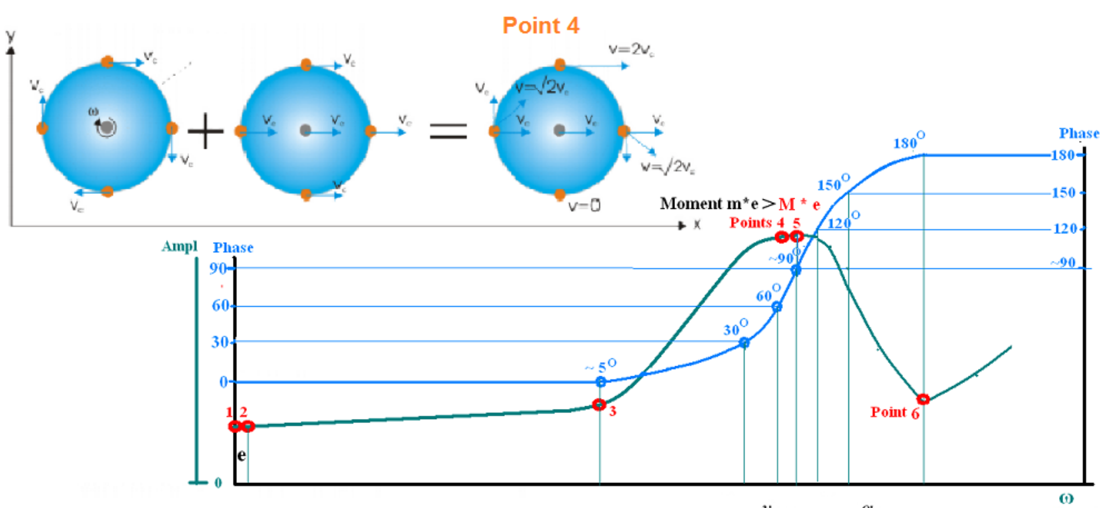
Figure 4 Proportionality equation of horizontally oriented continuous rotor constrained on rigid supports. Rotor fundamental harmonic resonance is excited when rotational angular velocity equals rotor's natural resonance frequency

B.1.b.1. Rolling without and with slipping- Shaft Centerline Path Establishing gravity stability treshold

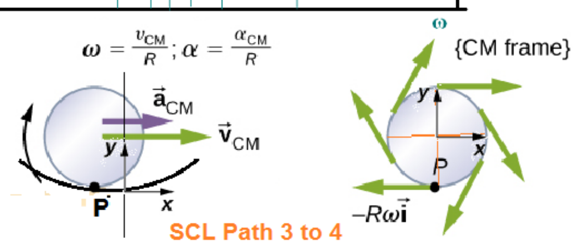
Energy is conserved in rolling motion without slipping. Energy is not conserved in rolling motion with slipping due to the heat generated by kinetic friction**.**

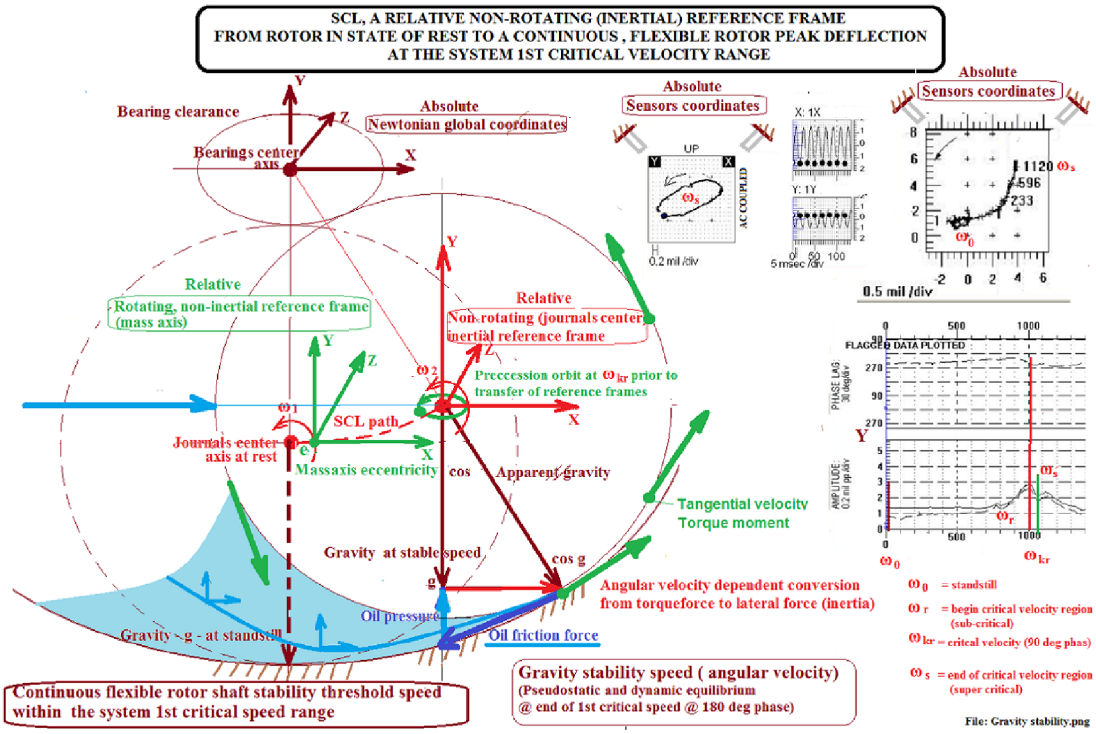
(Note: Figures for CW rotation)



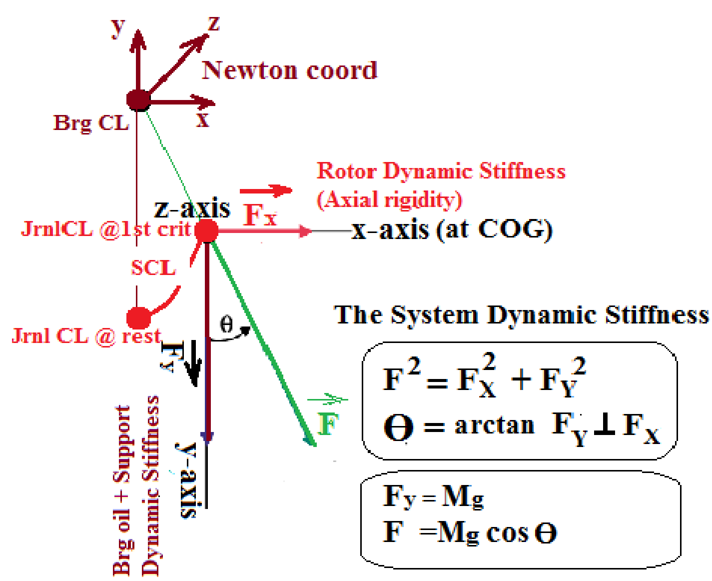


Shaft centerline path from subcritical (point 3-4) to critical velocity (peak at ~ 90 degrees)
and
Reaching stability treshold velocity and switching of reference frames between points 4 and 5





Rolling with slipping is a physical phenomenon that occurs when a rolling object (rotor journal) experiences both rotational and linear motion but also experiences frictional forces that cause slipping. This phenomenon can occur in various situations, and can be applied to rotor journal in oil bearings accelerated from standstill until reaches angular velocity and bearing load attitude angle **(Note: Figures for CCW rotation).**



Study has shown that "**1st critical speed** "of the system with "rigid" or "flexible" continuous rotor on elastic supports, the observed **forced frequency response** at system 1st critical speed, **is**

<u>not equivalent to rotor fundamental harmonic resonance frequency of rotor in free state in gravity environment</u>. The system "1st critical speed" is angular velocity region, which begins at point when rotor masses are accelerated by external torque, and the radial force moments, axially distributed along the rotor centroidal axis between gravity constraints (bearings supports), begin advancing in time, relative to deflected geometric rotor body , and increasing reactive moment of inertia, resisting forward momentum from torque to bring rotor in natural motion of rotation around centroidal mass axis. The reactive moment of inertia resists switch to natural motion while laterally orbiting at synchronous precession up to force leading rotor deflection at ~90 degrees. Both, rotor peak deflection and phase reference on shaft are referenced to Newtonian global coordinates [5,6,7,8,9,10]. A forced flexible rotor absolute displacement at peak critical velocity is dependent on rotor body material properties and mass axis eccentricities relative to rotor geometric neutral axis (see pg.24)
<u>In order to reduce the displacement of the excitation of a continuous rotor at its inherent fundamental harmonic resonance frequency is by minimizing mass axis eccentricity by machining in shop, or by "balancing" the rotor in minimum of three balancing planes on balancing machine to restore rotor's mass symmetry relative to journals centerline axis.</u>
<u>In order to avoid possibility of rotor operation at its natural resonance frequency, that is done by a proper tuning of dynamic stiffness of bearings and rotor gravity supports relative to rotor dynamic stiffness based on rotor material properties, to be 10-20% below 0.5 x wave velocity (in/sec).</u>

$$v\,(w) \propto \sqrt{\frac{\dfrac{T}{\frac{V\rho}{g}}}{L}} = \frac{in}{sec} \quad \text{(velocity of sound)}$$

> **A wave (W) can be thought of as a disturbance or subatomic mass particles oscillation that travels through space -time, accompanied by transfer of energy between systems. The direction a wave prpagates longitudinally is perpendicular to the direction particles oscillate for transferse waves. A wave does not move mass in the direction of propagation: it transfers energy between systems**

Figure 5 Proportionality equation of fundamental harmonic resonance velocity of sound of continuous rotor in free state externally unconstrained

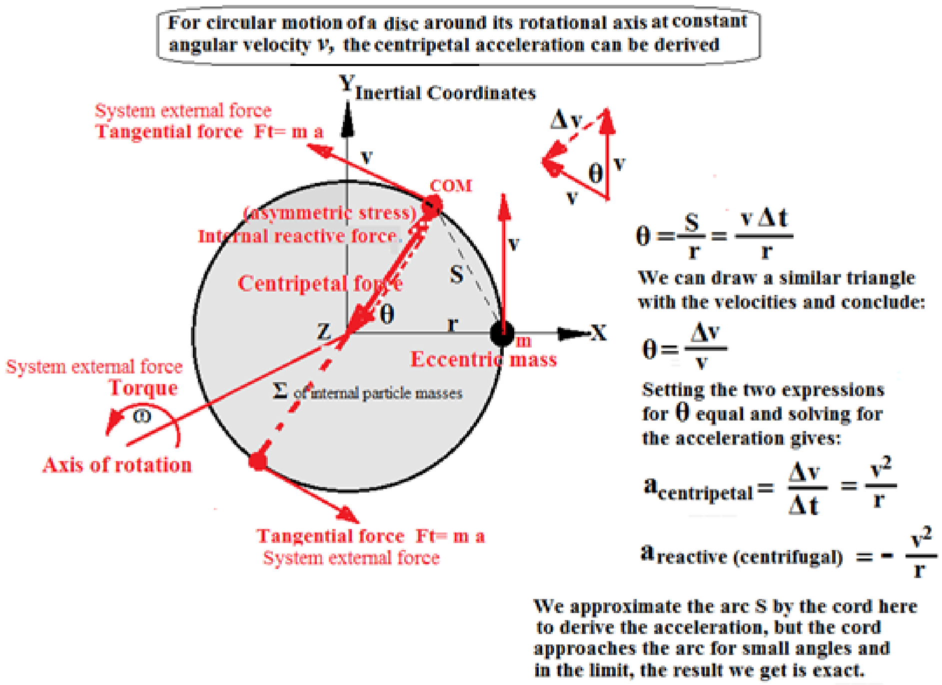
B.1.c Various States of Continuous Horizontal Rotors Constrained by Gravity on Various Bearings and Supports

B.1.c.1.- Circular motion

<u>A third law of rotational motion</u> states that **for every (<u>centripetal)</u> force pulling inward toward the center of rotation, there is and equal and opposite <u>(centrifugal)</u> force pushing outward from the center of rotation**.

CIRCULAR MOTION

Conversion of system external forces (torque)to circular motion, centripetal acceleration, and to internal stress (reactive centripetal force) in a disc , proportional to circumferencial velocity, referenced to rotating (non-inertial) frame.



On "continuous" , solid mass body rotor-shaft of finite length L ,when accelerated by external torque force, every constituent atomic particle has angular velocity , with instantenous tangential velocity which changes vector direction as it continue to rotate. By following Newton first and third laws of rotation, enrgy from external torque converts to rotor mass circular motion and centripetal acceleration, which on solid mass converts to rotor potential energy.

A third law of rotational motion states that **for every mass particle symmetrically centered around mass axis, (centripetal) force is pulling inward toward the center of rotation, and on each mass particle there is equal and opposite (centrifugal) force pushing outward from the center of rotation**. At some extreem angular velocity, centrifugal forces acting on each mass particle, will exceed material property electromagnetic binding energy forces and rotor-shaft will disintegrate. When rotor-shaft rotating in oil bearngs on elastic supports is constrained by gravity, it is preventing rotor with eccentric mass to revert to natural rotating motion around mass axis.

If mass eccentricities are radially asymmetric, forming an assymteric mass axis between gravity constraints (nodal points), centrifugal forces and axial moments will deflect rotor-shaft, with increasing deflection (pseudo static) , as it is accelerated through the system 1st "critical speed, reaching the maximum at angular velocity when centrifugal forces leads deflection (vibration displacement) in time by ~ 90 degrees angular phase. As rotor is further accellerated, within seveeral revolutions, inertia momentum overcomes forces of gravity constraints, and rotor-shaft reverts to natural rotation self-centered around rotor-shaft COM.

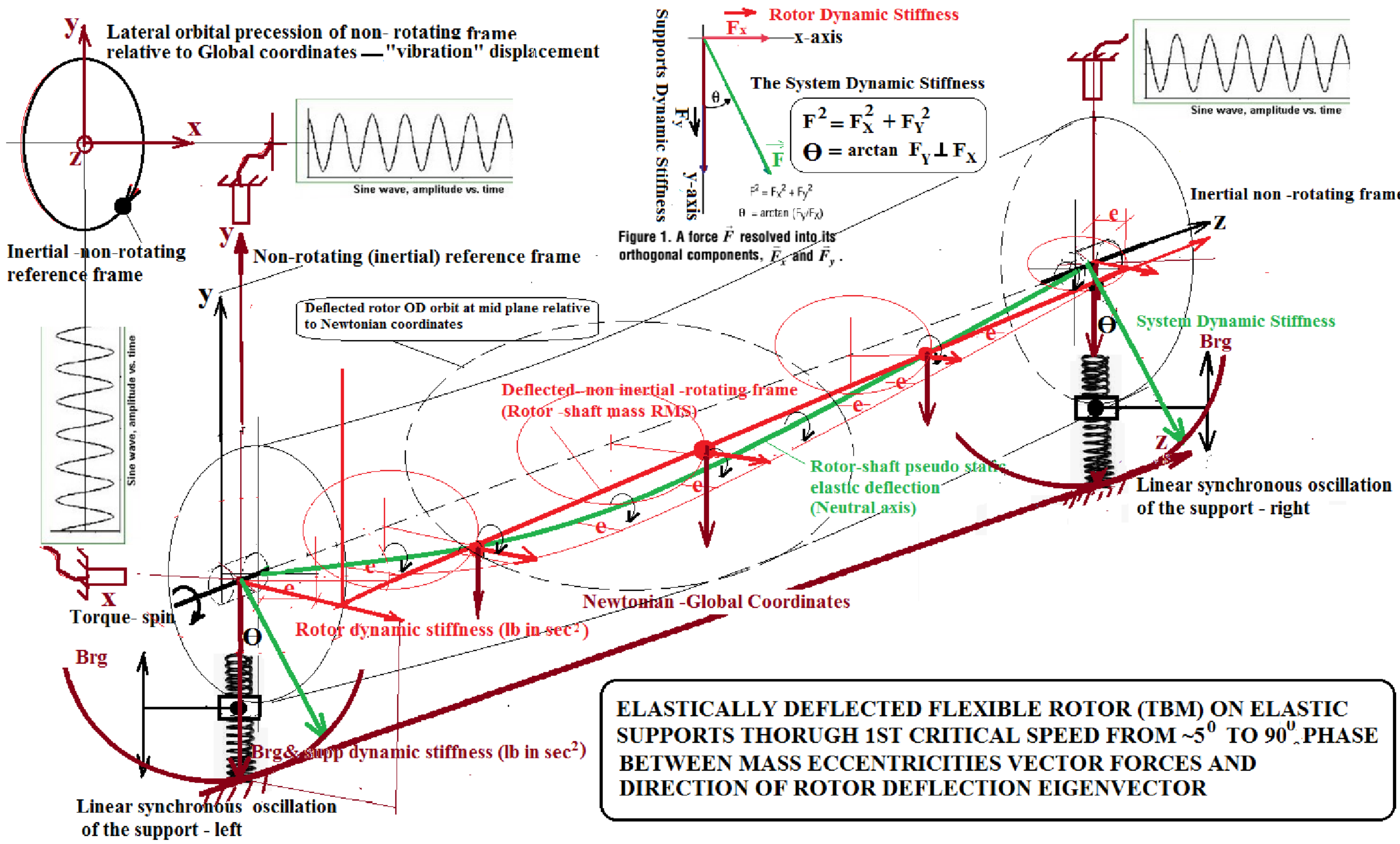


Figure 6. A flexible rotor with eccentric mass axis (TBM) on elastic supports at system "1st critical speed" shows rotor deflection against its dynamic stiffness perpendicular to gravity, and supports dynamic stiffness in direction of applied CF in direction of gravity.

Figure 6. shows that the angular velocity at the system "1st critical speed", the rotor deflection vector (displacement in space) and direction of rotor lateral moment of inertia $\mathbf{Ir^2}$ vector tangential to deflection are perpendicular. Rotor dynamic stiffness, and supports dynamic stiffnesses in direction of gravity, are orthogonal at 90^o. Their geometric sum yields the rotor "unbalance" forces acting on supports at an attitude angle Θ relative to gravity force.

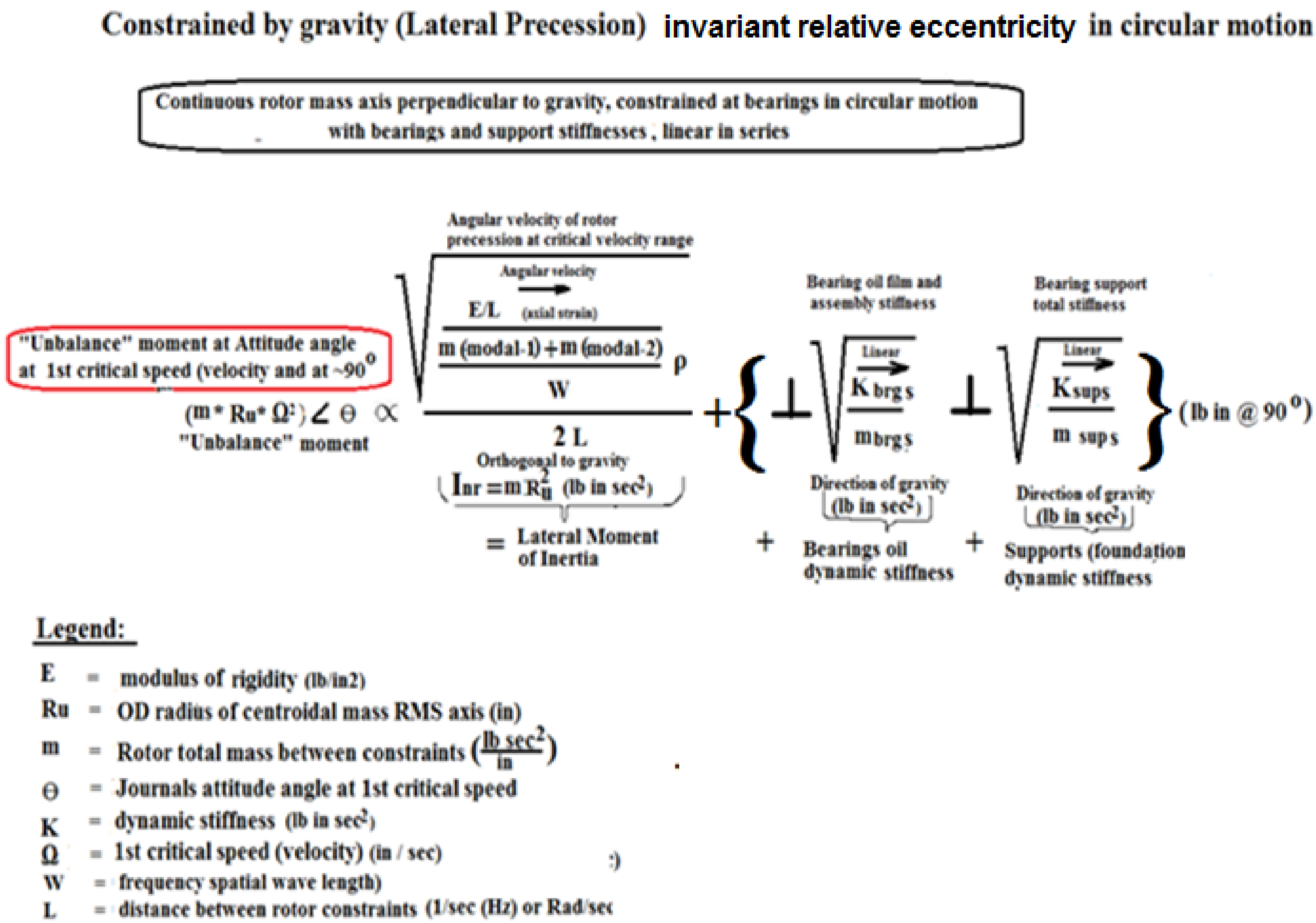


Figure 7. Equations for calculating the system 1st critical speed of continuous horizontally oriented rotor constrained by gravity on elastic supports. In Figure 8., the system 1st critical speed can be estimated by FEM modeling

1. Continuous Rigid Concentric Rotor at REST on Rigid Supports

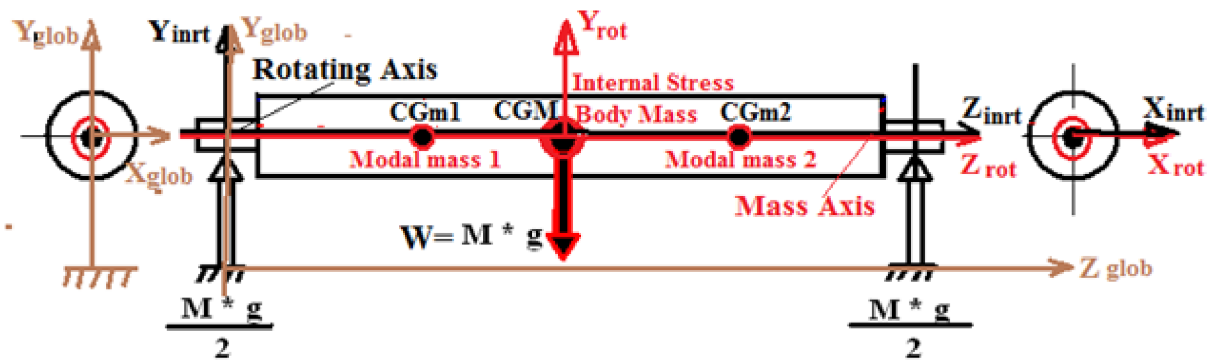


Figure B.1 **Continuous Concentric Rigid Horizontal , Gravity Constrained Rotor with Three Reference Frames; Rotor COG and COM are coincident**

Continuous rotor is considered a "body" or an aggregate of matter (mass). Its total energy is "potential" and it is a sum of modal masses. In this case the inertial axis and rotating axis are coincident. The centers of gravity CGm1, CGm2 and CGM are all coincidental forming a mass axis coincident with inertial Z-axis in all frames. Total energy of the rotor at rest is its mass (M) times gravitational acceleration "g".

2. Continuous Rigid Rotor with Axially Distributed Eccentricities on Rigid Supports

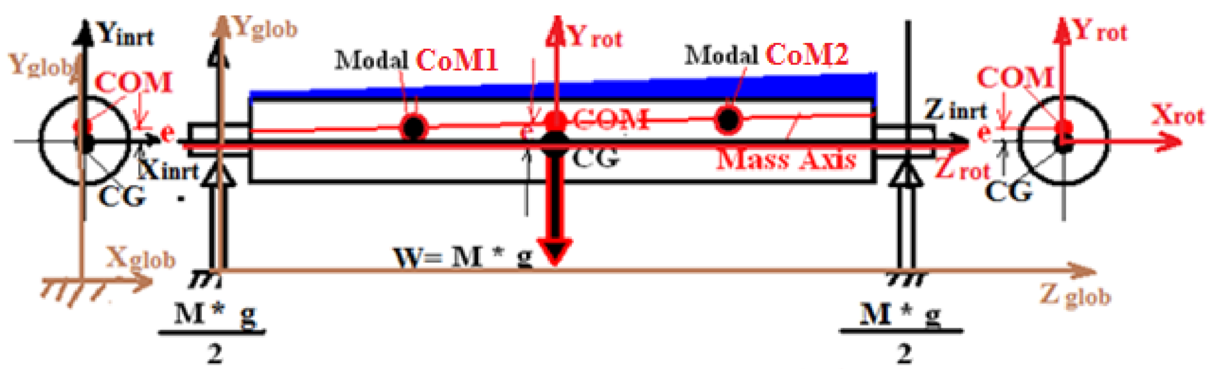
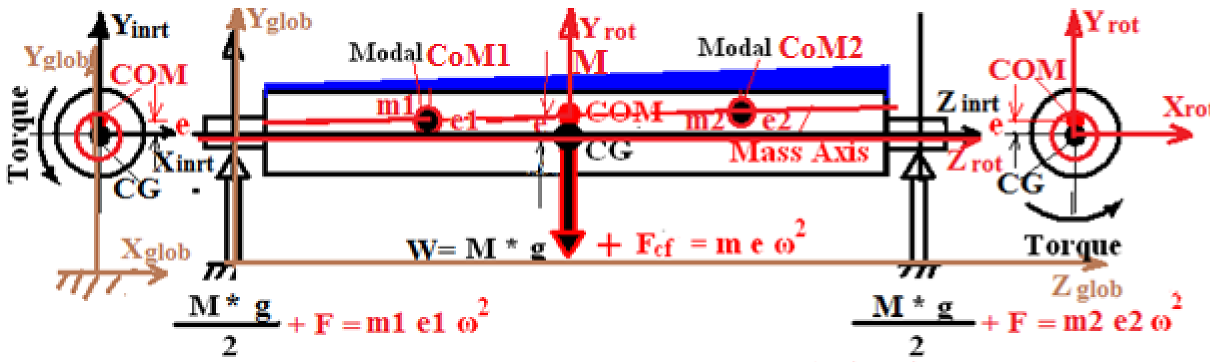


Figure B.2 **Continuous Eccentric Rigid Horizontal, Gravity Constrained Rotor with Three Reference frames; Rotor . In state of rest COG and COM are c oincident**

Whether rotor is concentric or eccentric at REST, its total energy is "potential" and it is a sum of modal masses. The rotor mass axis intersects the centers of gravity Modal CG1 and Modal CG2, and cross COM which is radially offset from rotating axis by eccentricity "e". Still a total energy of the rotor at rest is its mass (M) times gravitational acceleration "g".

3. Continuous Rigid Rotor with Axially Distributed Eccentricities, accelerated up to system fundamental Harmonic mode frequency



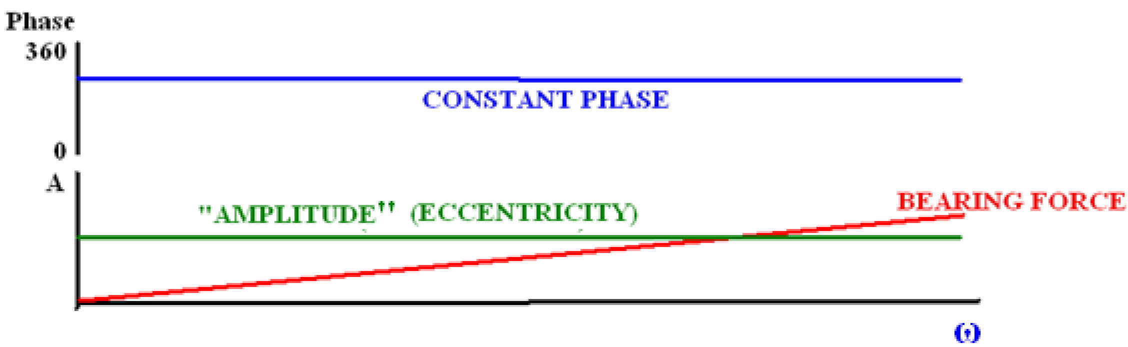


Figure B.3 Continuous Eccentric Rigid Horizontal, Gravity Constrained Rotor on Rigid supports with Three Reference frames; Torque is applied to move rotor mass from state of rest and accelerate it up to system fundamental harmonic frequency. Mass axis is **precessing** around rotational axis

When a continuous eccentric rigid rotor constrained by gravity on rigid supports is accelerated, mass axis precesses around rotating (journals centerline) axis. Centrifugal forces converted from torque (external energy input to system), torque moments (m1 *e1) and (m2*e2) generate cyclic forces at constraints. The effect of reaction forces are presented on a Bode plot. Additional (supplemental torque is required to overcome inertia from eccentric mass.

4. Continuous Rigid Rotor with Axially Distributed Eccentricities, accelerated above system fundamental Harmonic mode frequency

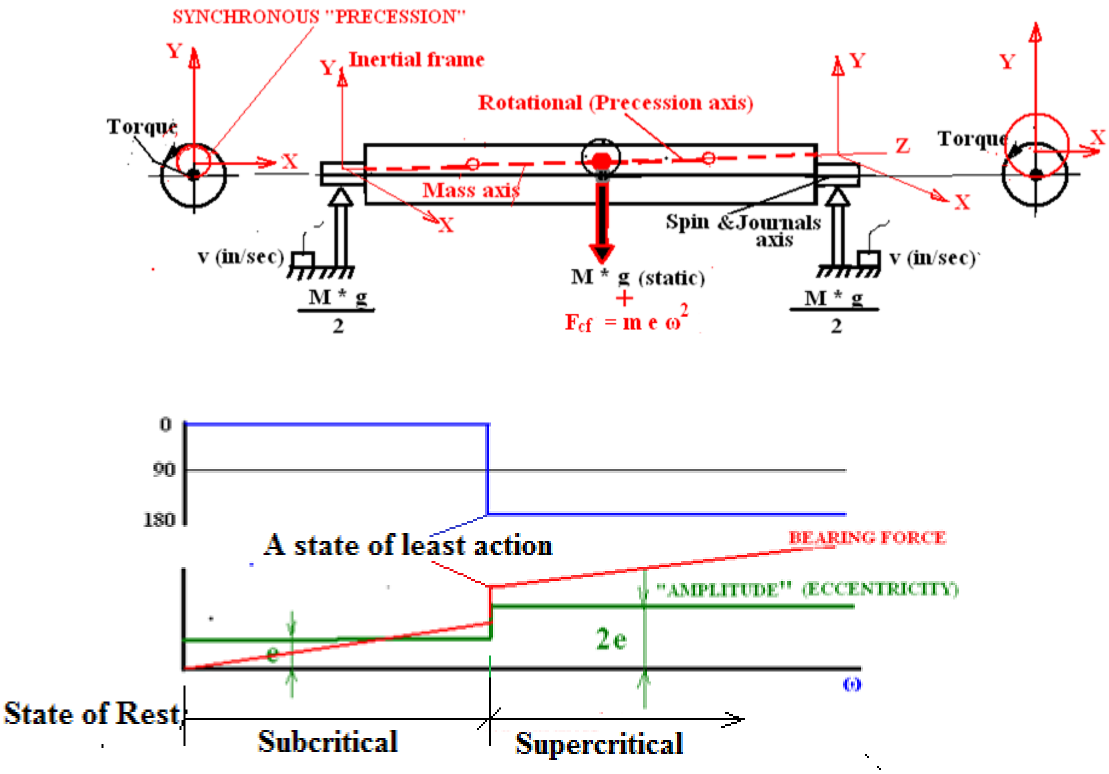


Figure B.4. Continuous Eccentric Rigid Horizontal, Gravity Constrained Rotor on Rigid supports with Three Reference frames; Torque is applied to move rotor from state of rest and accelerate it through, and above system fundamental harmonic frequency.

Rigid rotor driven by torque at center of non-centroidal axis above the system fundamental resonance frequency range, non-centroidal axis precesses around rotor's mass axis in natural motion, **self-centered by inertia** forces in space as defined by Newton 2^{nd} and 3^{rd} Law (Figure 4)

COM of axis is a constraint in space, and rotational axis is laterally translating around mass axis in lateral orbital motion. At phase angle of 180 degrees rotor reaches a state of least action: no torque is required for continuous rotation except for overcoming external forces (friction and aerodynamic).

B.1.d. FEM of Continuous Rotor Horizontally Oriented Constrained by Gravity

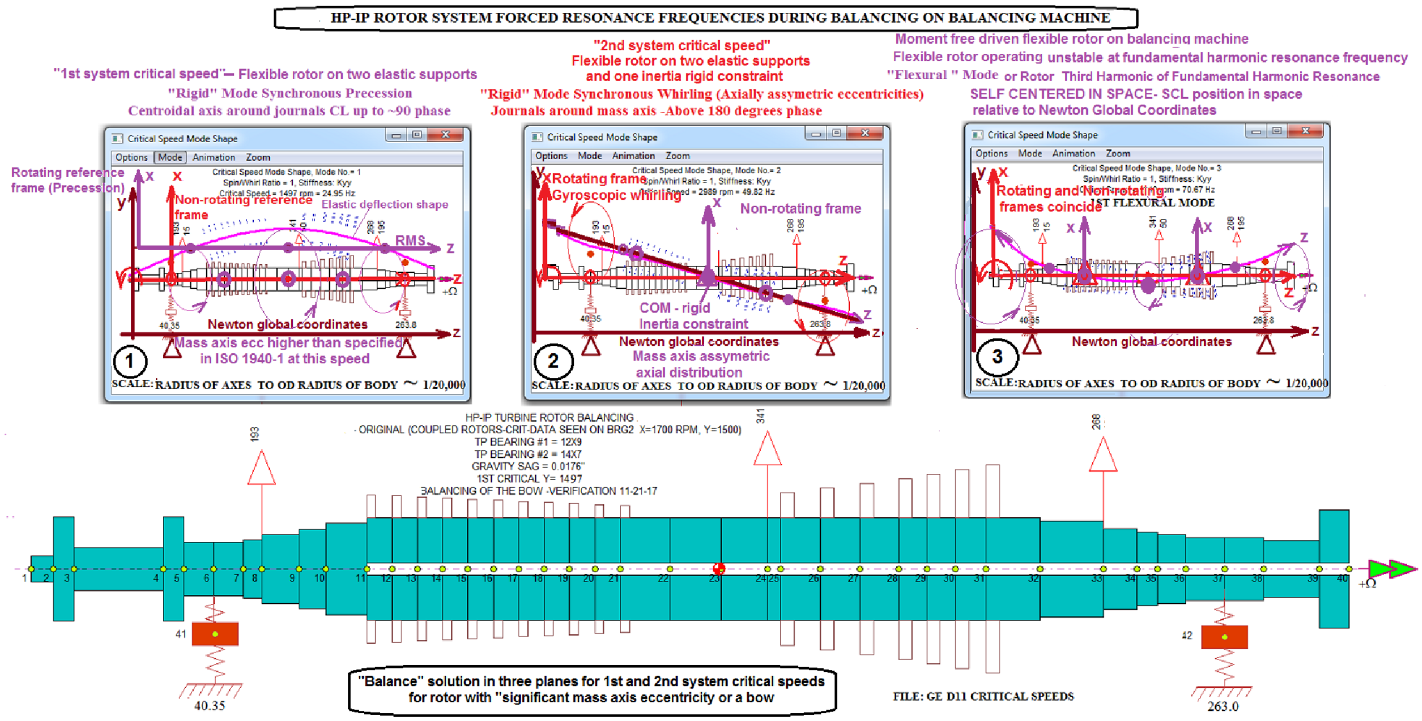


Figure 8. Calculated "rigid" modes of 1st and 2nd critical speeds of an actual HP -IP continuous rotor on elastic supports, by "tuning" a system stiffness of a flexible rotor , to oil bearings on elastic supports stiffness .

Figure 8. represent operating frequencies 1st and 2nd "rigid" modes of a statically deflected flexible real continuous rotor on elastic supports. The operating speed frequency, and the efficiency of an operating horizontal rotating machine consisting of “rigid” or “flexible” rotor, constrained by gravity in two bearings on elastic supports, within a finite “L” distance between gravity constraints, **must be lower than 0.5x of rotor natural resonance wave in free state**. In order **to avoid** the effect of unstable operation at rated speed frequency and load, operating speed of rotor must not be higher than fundamental harmonic resonance(first flexural mode), since any external pulsating torque or other excitation force, could destabilize pseudo static equilibrium of rotor (SCL) operating in oil bearings, and possibly excite the subsynchronous whirling (oil whip), when angular velocity equals 2x of the system 1st critical speed.

Graphic plot #1 in Figure 8 shows relation between **mass centroidal axis as rotating frame** relative to **journals CL axis as non -rotating frame** from state of rest up to peak deflection at ~ 90 degrees precessing **through the system 1st critical velocity range**[14,17]

Precession magnitude of elastically deflected rotor is dependent ,and proportional to rotor angular velocity and centroidal mass axis eccentricities radial offsets.

Graphic plot #2 shows relation between **mass centroidal axis as non-rotating frame**, and **journals CL axis as rotating reference frame** after peak deflection above ~ 90 degrees, as "2nd critical speed mode". In this mode both journals are whirling in gyroscopic motion, referenced to inertia constraint at rotor COM, self centered in "space", following Newton law of mass symmetry in rotating motion. The "2nd critical speed mode" is resulting from **axial asymmetry** of mass distribution within each of two modal elements, relative to rotor COM, along rotor centroidal mass axis between bearings gravity constraints. Journals whirling magnitude dependents and is proportional to rotor angular velocity and residual masses axial asymmetric distribution between constraints and corresponding axial moments.

Graphic plot #3 in Figure 8 shows rotor amplified displacement response at fundamental harmonic resonance frequency excited by asymmetric torque. Maximum rotor operating frequency must be 10% to 20% below its fundamental harmonic resonance frequency, for rotor stable operation.

B.1.e. Practical Calculation of Rotor 1st Critical Speed

The 1st critical speed of horizontally oriented flexible rotor in **state of rest**, can be calculated fairly accurately using rotor as Euler beam, for specific operating speed using the known and measured physical rotor geometric parameters, based gravity acceleration acting on rotor mass as "excitation force".

Practical Calculation of rotor 1st Critical Speed

Based on known and measured parameters

RPM = Calculated 1st critical speed

W = Known (lbf)

W = M * g (lbm)

δ = Gravity sag measured (in) or calculated from FEM model)

0

$$f_n = \sqrt{\frac{K}{M}}$$

$$RPM = \frac{60}{2\pi}\sqrt{\frac{K\ lb/in * 386\ in/sec^2}{M\ lb}}$$

$$RPM = 187.6\sqrt{\frac{K}{M}}$$

$$K = M\left(\frac{RPM}{187.6}\right)^2$$

$$RPM = 187.6\sqrt{\frac{1}{\delta}}$$

For stability of machine in operation, each of bearings and supports linear stiffness must be less than 50% of rotor's radial stiffness calculated from measured gravity sag

B.1.f. Continuous rotor fundamental resonance vs. system 1st critical speed [27]

For a stable rotor operation at rated speed and load, in rotating machine system, the total system dynamic stiffness resultant is addition of two perpendicular vectors i.e. of rotor internal radial dynamic stiffness based on material atomic property in quantum field, and bearing oil and bearing support dynamic stiffness in gravity field. And for rotor stability at system 1st critical velocity, rotor tangential velocity must be at least 10% to 20% lower, than 0.5x frequency wave velocity (eigenvector over continuous rotor length "L") of rotor natural harmonic frequency wave "W" in free state.[19]. At the same time the RMS value (centroidal mass axis) of deflected rotor's radial deviation (e + d) distributed axially along rotor length "L", and journals axis must be brought to symmetry ("balance”).

That can be achieved by restoring the symmetry between centrifugal forces generated from static eccentric centroidal mass axis laterally precessing moment of inertia I r^2 of area L X D, (a.k.a. unbalance), and dynamic mass axis created from correction masses, proportionally distributed in a minimum of three planes axially on rotor body two rigid modal elements, between elastic supports constraints.

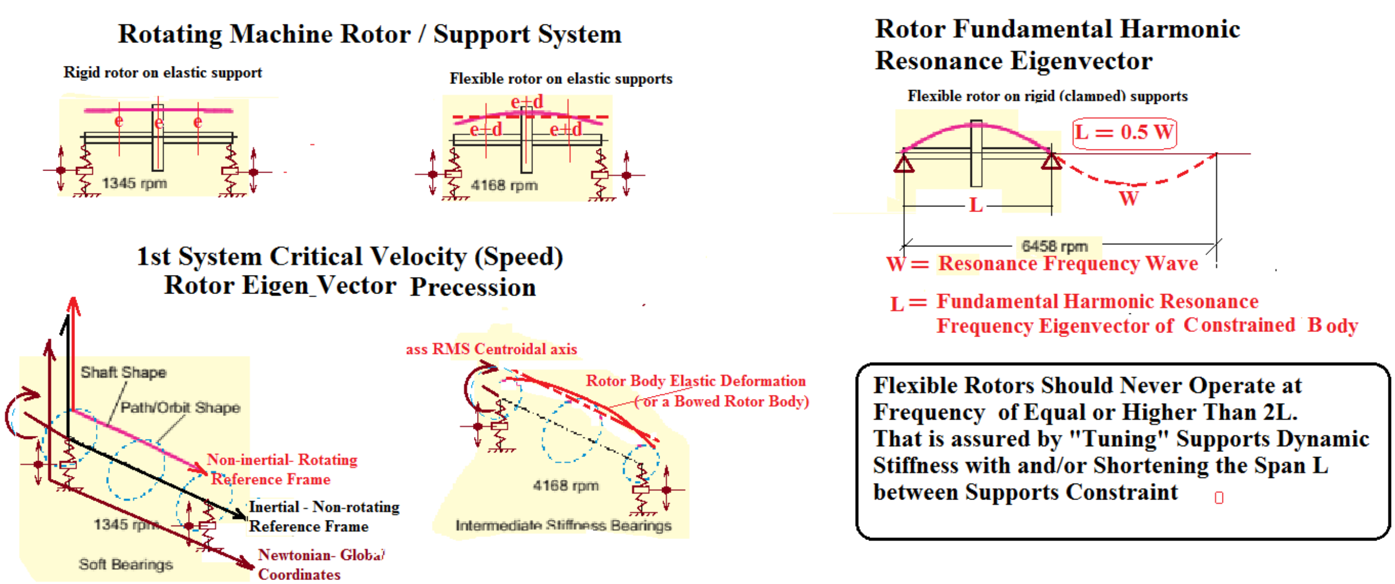


Figure 9. Rotating machines with rotor and supports as an open system (Modified sketches from reference [27].

B.1.g Determination of continuous rotor allowable residual eccentricity to minimize "unbalance" responses and maximum operating speed

Balancing of a rotor which has eccentricities (i.e. runouts measurable with a dial indicator) is generally accepted in industry as a tradeoff between reducing 1X forces transmitted to ground through bearings, seals and so forth, and reducing 1x displacement amplitudes at important locations.

When discussing a tradeoff between reducing 1X forces transmitted to ground through bearings, seals and so forth, and reducing 1x displacement amplitudes at important locations is resulting from "balancing" relative to different reference frame. Although it is still acceptable as a tradeoff on majority of smaller and high speed rotors with relatively high ratio of power over rotor mass, on systems with a variety of bearings and support types in rotating machinery, that is not the case when balancing large turbine and generator rotors with large moment of inertia and operating at relatively low operating speed, where stable machine operating speed is highly dependent on various physical parameters as shown in table below.

ROTOR TYPE		BEARING TYPE		SUPPORT TYPE		ROTOR F-RES	SYSTEM 1ST F-CRIT	ROTOR F-OPER
RIGID	FLEX	RIGID	OIL	RIGID	ELASTIC	ω_{res}	Ω_{crit}	ω_{oper}
X		X		X		HIGH	HIGH	$\omega_{oper} < \omega_{res} < \Omega_{crit}$
X			X		X	HIGH	$\Omega_{crit} < 0.5\ \omega_{oper}$	$\omega_{oper} < \omega_{res}$
	X		X		X	LOW	$\Omega_{crit} < 0.5\ \omega_{oper}$	$2x\,\Omega_{crit} < \omega_{oper}$
	X	X			X	LOW	$\Omega_{crit} > \omega_{oper}$	$\omega_{oper} < \Omega_{crit}$

Figure 10. Parameters interdependence of rotating machinery stability

From the above table we can see that the design operating speed, and a stable operation at maximum design efficiency is dependent on rotor atomic material properties (atomic crystalline structure dynamic stiffness, and L/D ratio, determining the continuous , semi-infinite rotor's fundamental harmonic resonance frequency in free state. The type and the type of bearings and supports, when such rotor is horizontally constrained on rigid or elastic supports determines the system 1st critical speed. The magnitude of permissible vibration forces, and journals displacements , whether rotor is "rigid" or "flexible" (deflected by gravity in state of rest), are dependent on rotor centroidal mass axis eccentricity relative to journals rotational axis.

The critical parameter for rotor stable operation in rotating machine as a system, is a determination of the system 1st critical speed relative to design operating speed. This is achieved by "tuning" gravity supports dynamic stiffness and rotor material inherent dynamic stiffness orthogonal to gravity. These two dynamic stiffnesses vectorially added yield the system dynamic stiffness and the attitude angle in direction of rotation at the peak of system 1st critical speed at 90 degrees phase.

At the end of 1st system critical speed velocity range, and at 180 degrees phase between the rotor internal torque- eccentricities axial force moments , and accelerated external physical rotor body pseudo static deflection, rotor reaches a state of minimum action and dynamic and pseudo static equilibrium.

With "rigid rotors the angular change from 0 to 180 degrees occurs practically instantly. With flexible rotor on elatic supports, the change from 0 to 90 and 180 degrees occurs over a angular velocity range, the range of velocities is proportional to rotor dynamic stiffness over support dynamic stiffness and it is indicated by so called "amplification factor".

A goal for any turbine or generator rotor in operation is to balance rotor mass to minimum practically achievable eccentricity relative and referenced to journals centerline axis, and thus reduce transmitted forces to bearings when accelerated to the system design operating speed.

In order to standardize the allowable mass axis eccentricity of rotor COG, which is coincident to COM of rotor assumed as a point mass in state of rest, International Standard ISO 1940-1 was developed as reference exclusively for rigid rotors since eccentricities are invariant regardless of rotor angular velocities, and forces generated are proportional strictly to square of operating speed, so that the total 100% of potential energy from "unbalance" forces is considered to be transmitted to bearings. With previous assumptions, maximum allowable eccentricity is converted to maximum permissible residual "unbalance", forces in bearings for specific type of rotating machine.

A study has shown that even for a flexible rotors with eccentric mass the allowable maximum mass axis eccentricities can be extracted from ISO 1940-1.

If we assume a horizontally oriented flexible rotor supported on supports while in state of rest, gravity acceleration will cause rotor mass to deflect mimicking 1st mode eigenvector. If we calculate the RMS value of deflected rotor, it will represent all axially distributed "unbalance" vectors as rotor eccentric masses axially distributed along centroidal mass axis. If we further assume that 1st mode eigenvector consists of two modal elements, rotor body centroidal rotor mass axis will intersect rotor geometric neutral axis at two places which we can considered COMs of two modal elements, which sum would equal total rotor mass.

When an eccentric flexible rotor on elastic supports is accelerated through the system 1st critical speed (up to 90 degrees phase), a total rotor body deflection and its RMS value will be at its maximum response values at measuring points at journals. If we take the speed at which maximum deflection occurs, we can determine from the ISO1940-1 graph maximum allowable rotor eccentricity

From this it can be concluded that even in a case of determining the allowable residual eccentricity of a flexible rotor, the ISO 1940-1 can still be applicable. But instead of referring to design operating speed of rotor in diagram (Figure 14), the speed at maximum fundamental resonant frequency response should be used to determine the acceptable residual eccentricities of the fundamental mode modal elements of body mass axis in a quiescent state. Based on a new way of interpreting the ISO 1940-1 diagram, it can be concluded that a root cause of “unbalance” at specific rotor speed (rev/sec) is rotor’s body mass axis radial distance and slope in space relative to rotor rotational axis connecting two journals centers. The recognition of these facts creates a ground for the new definitions of “unbalance” and a base for developing new “balancing” methods of rigid and flexible rotors on balancing machines and in practice on operating machines [13].

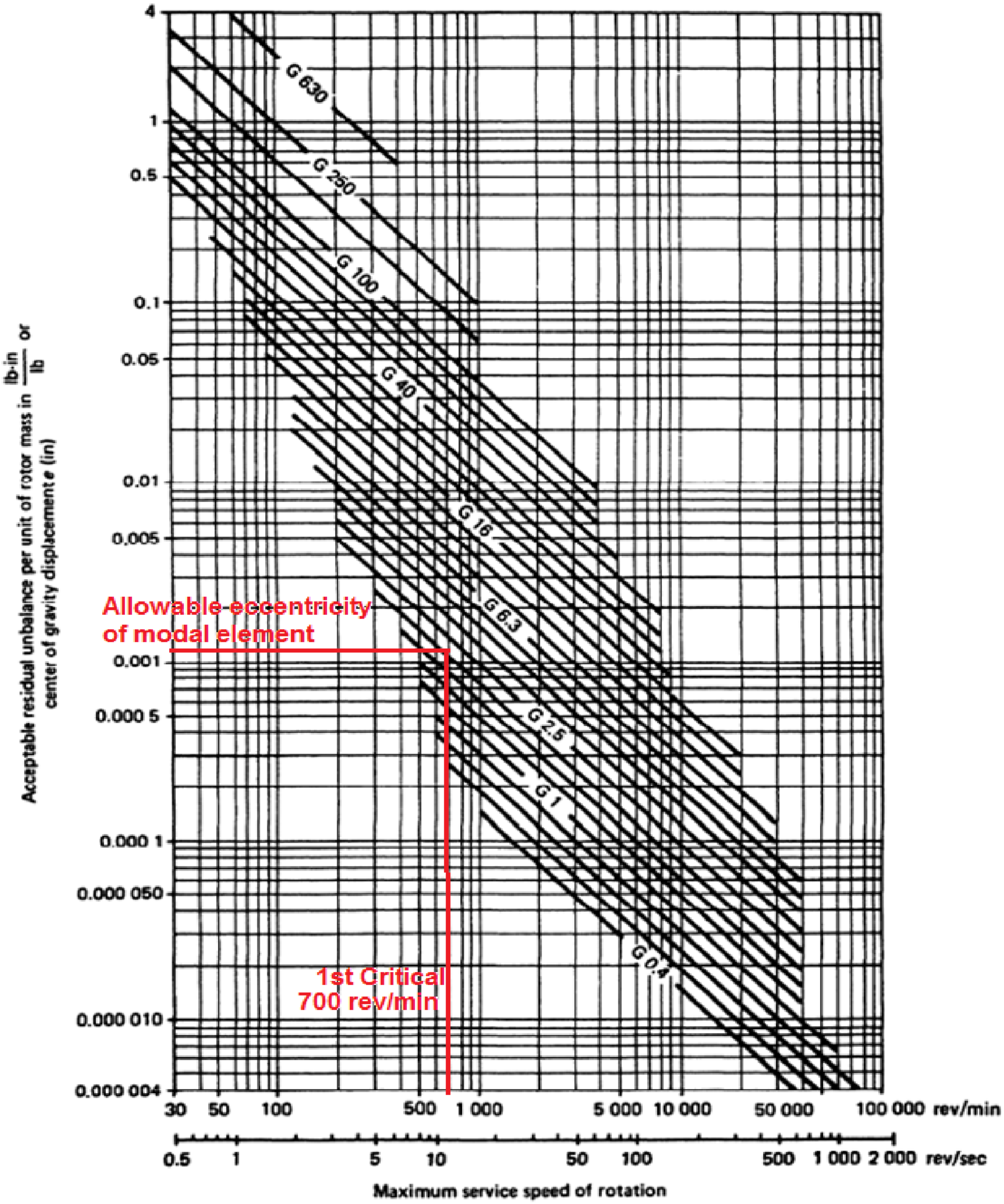


Figure 10. A new interpretation of “Diagram of the acceptable residual center of gravity displacement “e” of rigid fundamental mode modal element, vs. residual balancing quality at rotor service speed on balancing machine (mm/sec, in/sec-pk., on bearings, or journal displacement in μm p-p, or mils (0.001”) p-p of journal.

B.1.g.1 Flexible rotor-shaft operating speed and power limitations

Fundamental rule for determining the rotor stability at operatng speed for rotating machinery maximu obtainable eficiency is that operating speed at machine maximum process load MUST be lower than rotor fundamental resonance frequency in free-free state! the system 1stcritical speed eigen frequency MUST be 10-20% lower than 50% of rotor-shaft fundamental eigenfrequncy in free-free state. That is achived by tuning bearings and supports linear stiffness to be below rotor-shaft axial rigidity [30].

ROTATING MACHINE OPERATING SPEED AT MAX EFFICIENCY FREQUENCY WAVE (W)
MUST BE LESS THAN 2X 1ST CRITICAL FREQUENCY WAVE (L)

$$\omega_{(oper)} < 2x\, f_{(1ST\ CRIT)}$$

One Dimensional Longitudinal Propagation Wave (W)

@ angular velocity

@ Linear brg oilstiffness + Linear support stiffness in direction of the apparent gravity

(axial strain)

$$f_{(RES)} \propto \sqrt{\frac{\dfrac{G\left(\frac{\tau}{\gamma}\right)}{M\left(\frac{V\rho}{g}\right)}}{L}}\left(\frac{in}{sec}\right) \neq f_{(1ST\ CRIT)} \propto \frac{\sqrt{\dfrac{\dfrac{G\left(\frac{\tau}{\gamma}\right)}{M\left(\frac{V\rho}{g}\right)}}{L}}}{2L} (+\perp) \sqrt{\frac{K_{brg}}{M_{brg}}} + \sqrt{\frac{K_{sup}}{M_{sup}}} \left(\frac{rad}{sec}\right)$$

LEGEND:

- **G = Shear Modulus of Elasticity-Modulus of Rigidity**
- **M = Rotor Mass**
- **L = Rotor Length between End Nodes**
- **Δ L = Shear Longitudinal Elongation between Particles Faces**
- **V = Rotor Volume**
- **ω = Resonance Velocity of Sound**
- **τ = Shear Stress**
- **γ = Unitless Measure of Shear Strain**
- **ρ = Material Density**
- **σ = Tensile Stress**
- **ϵ = Tensile Strain**

File: TOTAL ALL GRAPHS /1z-EQUATION

B.1.h. Continuous rotor on elastic supports accelerated through system 1st critical velocity region [14,17]

1. Physics of resonance effect

In the theory of rotor dynamics the “critical speed” resonance is considered a phenomenon which is similar to resonance phenomenon in theory of oscillations. Appearance of resonance is explained by coincidence of rotational velocity of “rotor-shaft” system with rotor’s natural resonance frequency. However, the “critical speed” in rotor dynamics considerably differs from resonance of oscillation processes. “Critical speed” is a resonance phenomenon that creates conditions for final rotor’s **precessional axis switch from journals geometric axis to rotor’s mass** axis. As result, rotor and shaft which were rotating around precession axis with their heavy side outside up to resonance, after resonance they rotate with their “lightweight” side outside. In

simple words, one cycle of resonance is represented with two peaks and phase shift path from +90 through 0 to -90 degrees (total of 180 degrees), while “critical speed” is phase shift path is from 0 to 180 degrees with only one peak, best presented as Nyquist plot.
The most important physical event not recognized in current balancing methods based on classical mechanics is that non rotating frames change at the peak of maximum displacement amplitude of rotor/support system at velocity corresponding the phase shift of ~90 degrees referenced to Newton global coordinates [14,17]. The magnitude of rotor deflection at the peak of critical speed velocity range is dependent on "e" (mass axis eccentricity) relative to rotor constrained by gravity geometric deflection "d", rotor internal dynamic stiffness based on material atomic structure (lb in sec^2), and geometric dimensional ratio between L/D between gravity constraints.

2. Physics of continuous flexible rotor on elastic supports

When continuous flexible rotor is accelerated above supercritical velocity, mass axis (which was a non-rotating reference frame up to ~ 90 degrees phase) becomes a rotating reference frame and a gravity stability axis (SCL), self centered in space (within bearings and seals clearances), from >~90 degrees to 180 degrees phase. Dynamic orbit centerline (non-rotating frame - inertial stability axis through supercritical angular velocities, and a machine load increase at constant velocity), is **continuously self-centering*** in space, forced by inertia forces from the residual radially asymmetric and axially distributed mass eccentricities pivoting at rotor COM as a rigid constraint [14,17]. Journals centers during mass axis self-centering at same time are synchronously whirling as gyroscopic horizontal rotating pendulums, rigidly pinned at COM "in space" at specific speed identified on SCL path, and flexibly constrained at respective bearings and elastic supports, with the size of orbit proportional to CFs from torque moments at particular rotor angular velocity. Whirling motion is observed at journals, in three degrees of freedom, linear in X and Y and rotation around Z axis, referenced from sensors installed relative to Newtonian global coordinates [13,15,16].

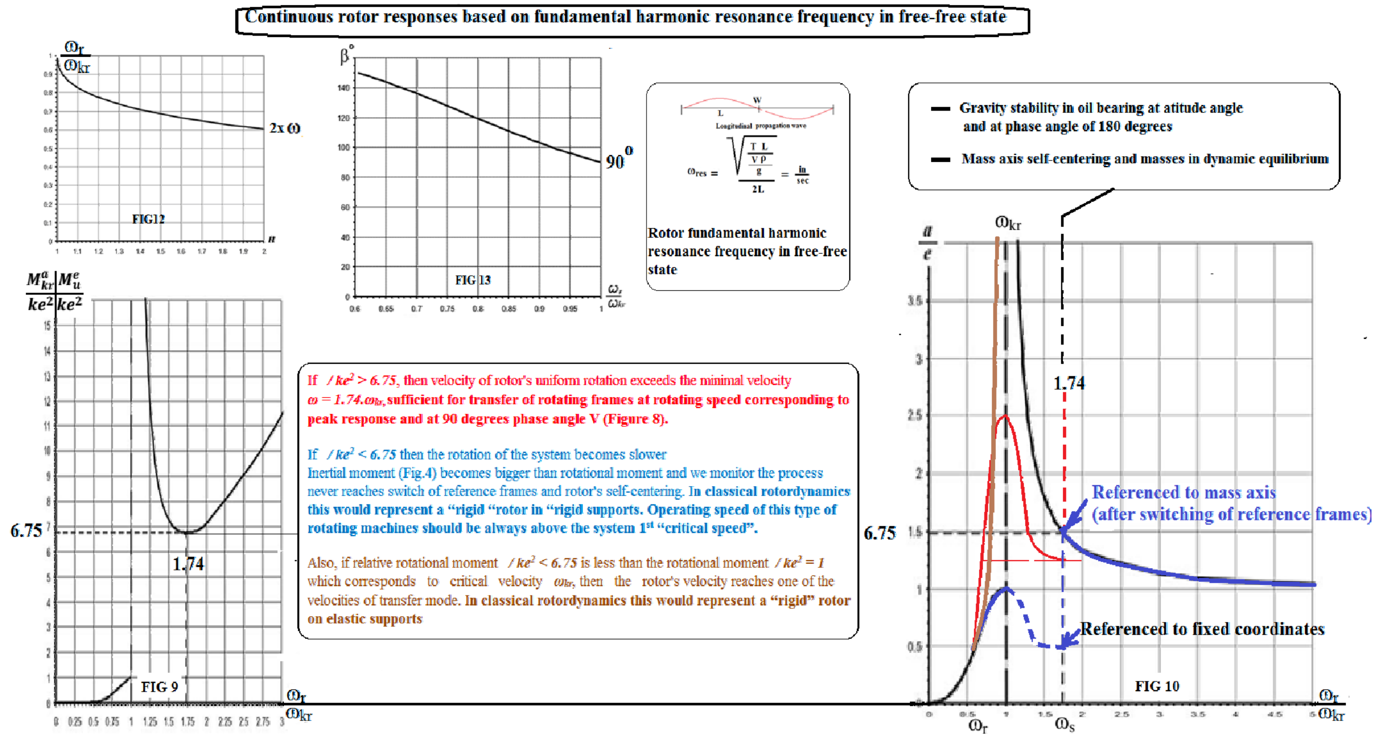


Figure 11. Summary of charts from bibliography reference [17] *

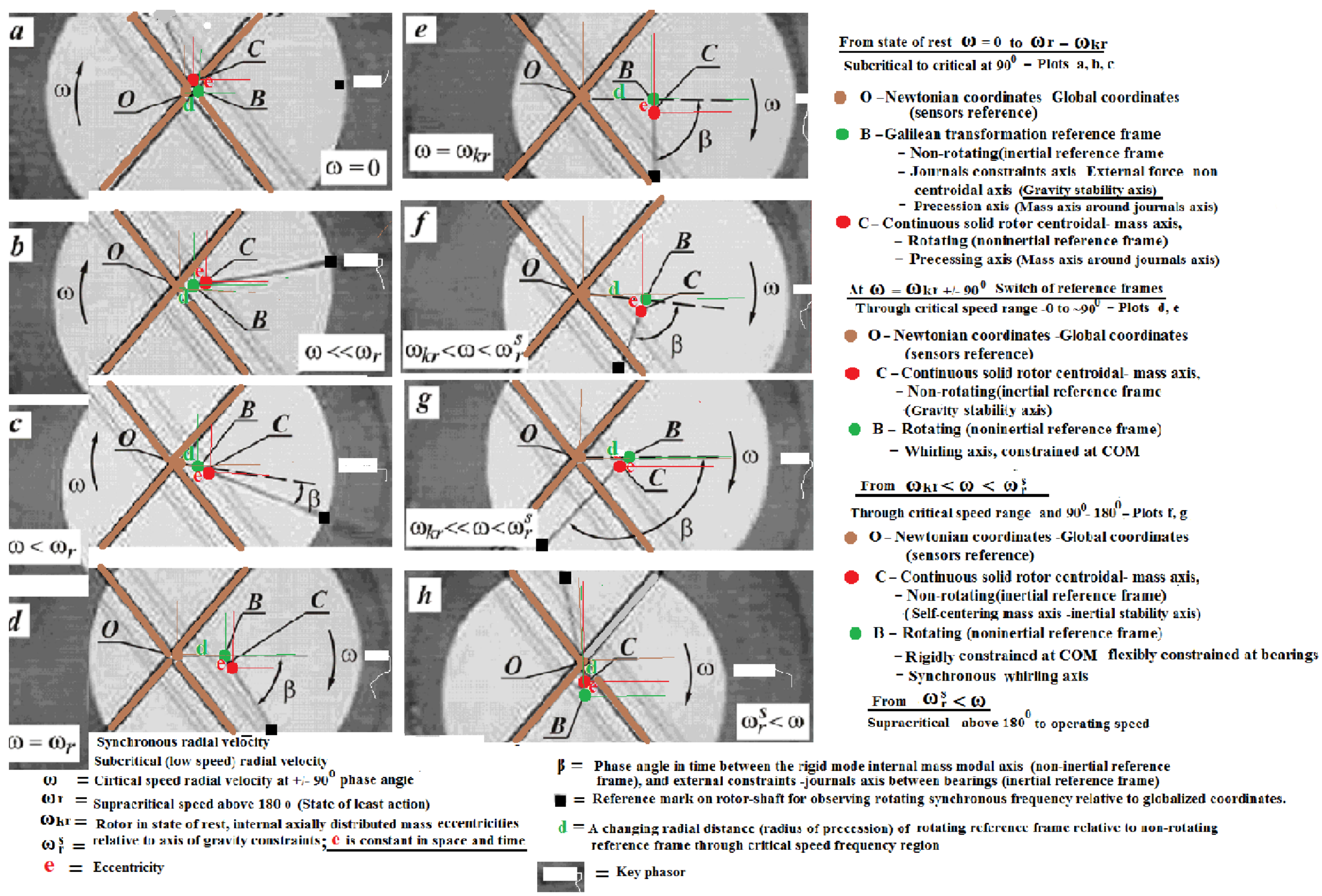


Figure 12. Video snapshots from an experiment monitoring spatial changes between rotating (non-inertial) and non-rotating (inertial) reference frame relative to Newtonian global coordinates from state of rotor at rest through critical velocity range points at subcritical, critical and supercritical velocities. *

* Graphs in figures 11 and 12 are extracted from reference papers [14 and 17] for reference only, because of .space constraint due to complexity of topic. For all details it is necessary to read full papers [14 and 17] in Bibliography.

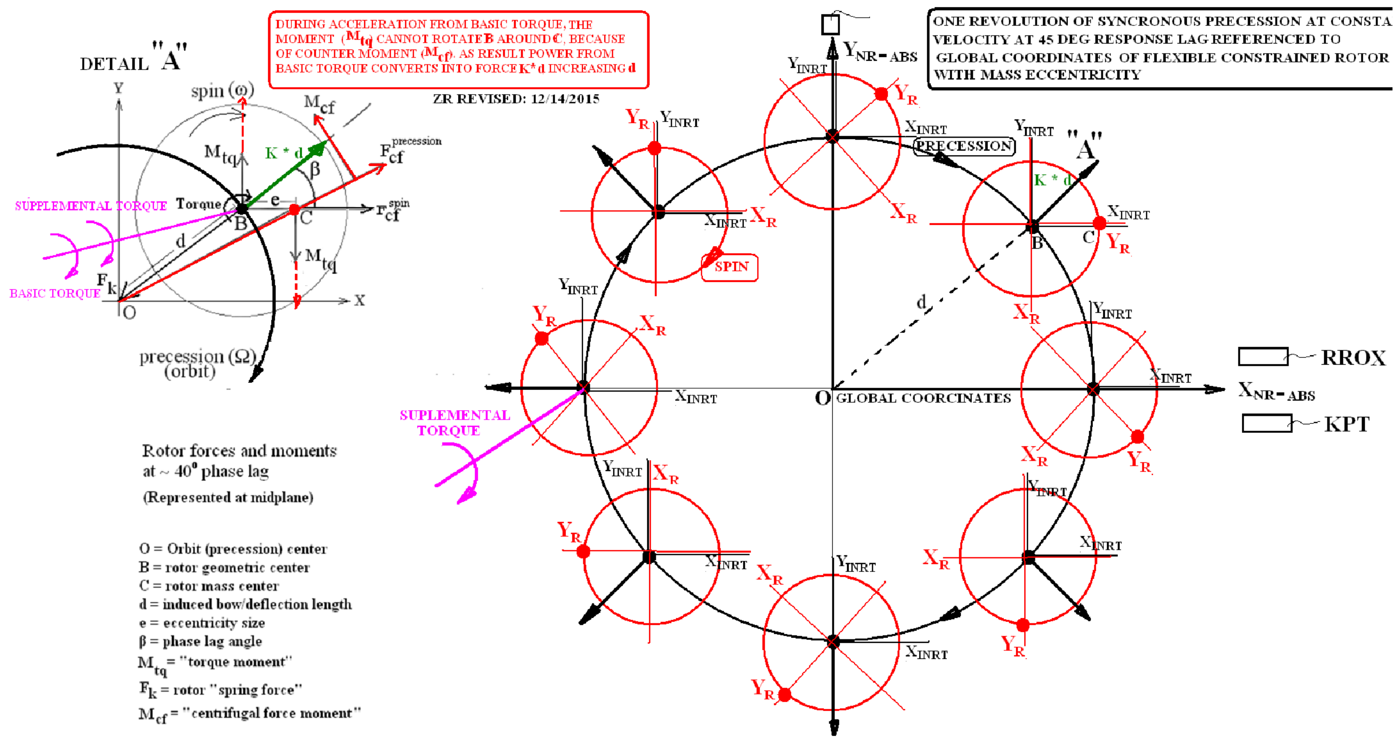


Figure 13. Reference frames in open rotating system

C. "Quasi-High Speed Balancing Method" on High Speed Balancing Machines

The objective of balancing rotors with mass centroidal axis (RMS value of deflected rotor body at system 1st critical speed), based on previous sections in the paper, is to vanish bearings dynamic reaction forces and displacements at journals, and to reduce the radius of orbital motion of eccentric rotor body between constraints (bearings) at criticals and at operating speeds, to the magnitude of internal, inherent, continuous body mass eccentricities, [3,4,12]. Whirling displacement amplitudes of eccentric, or a bowed body between constraints could be acceptable in practice, limited by design clearances value between rotating and non -rotating seals, but at cost of ultimately affecting machine process operating *efficiency.*

Accordingly, **balancing methods of large turbine and generator rotors should be balanced, not based on the assumed "modal" linear oscillating point mass "unbalance response vectors" at antinodes, at 1st critical speed, but based on compensating the inherent body longitudinal "rigid" centroidal mass RMS value of elastically deflected rotor between bearings constraints. Using a new ,low speed balancing method QHSBN at subcritical speed, placing simultaneously in phase, in three axially selected planes, and the magnitude of correction masses proportionally axially distributed along the body length "L" between bearings constraint, resolves the vibrations and orbital dynamic motions measured at journals at system 1st critical speed and 2nd critical system speeds. Rotor "balanced" by this method will be also successfully balanced for all rotor speeds. By this balancing method <u>dynamic forces of correction masses,</u>** axially distributed in three planes, placed on OD of geometric body, are **<u>forming a dynamic mass axis</u>** mirror imaging the centroidal mass axis from rotor inherent eccentricities, which sum create a virtual body mass axis (non-inertial, rotating reference frame) which coincides with journals centerline axis (inertial, non-rotating reference frame) vanishing forces in bearings and dynamic displacements responses relative to Newtonian global coordinates (vibration sensors reference frame), yielding:

$$\left(\sum \mathbf{F}_{ecc} + \sum \mathbf{M}_{ecc} \right) - \left(\sum \mathbf{F}_{dy} + \sum \mathbf{M}_{dy} \right) = 0$$ (See Figure 18)

(Randomly and asymmetrically distributed axially) (Proportionally distributed in three in three axially selected planes

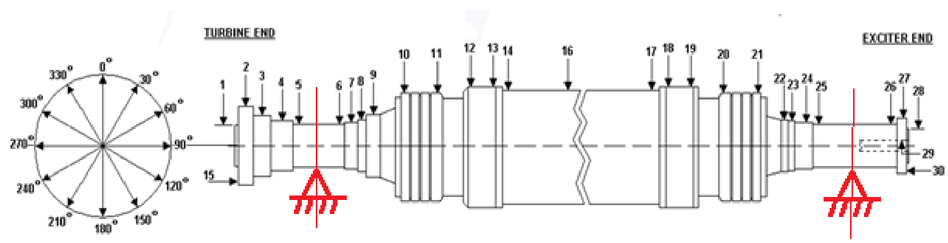
<u>Rotor will be "balanced at any speed!</u>

D. Rotor Mechanical Evaluation Prior to Balancing

D.1. Evaluation of Static Rotor Runout (eccentricity) by FFT

Static measured runout is that part of the total continuous rotor runout that is due to synchronous structural (and sometimes electric) sources, and does not change with rotational speed (and is thus **not a function of unbalance**). This is important to keep in mind when rotors with a bow or significant mass eccentricities are "balanced" based on readings by displacement sensors referenced to global coordinates with the **assumption** that the 1^{st} *system* critical frequency is equivalent to the fundamental harmonic resonance frequency of the rotor alone.

RUNOUT ALLOWANCES FOR STEAM AND GAS TURBINE AND GENERATOR ROTORS
OPERATING IN OIL BEARINGS AT 1800 RPM, 3000 RPM, 3600 RPM AND 5800 RPM



Rotor runouts can be measured using dial indicators or noncontact probes placed horizontally.
Rotor must be unconstrained, on lathe on prism blocks supported at journals CL and driven moment free

MAXIMUM ALLOWABLE TOLERANCES FOR ASSEMBLY AND ALIGNMENT

POINTS: 1, 15, 28, 29, 30 ....0.0005" (0.0127 mm) DIA

POINTS: 7, AND 24 ....0.0005" (0.0127 mm) DIA
(AS SENSOR MEASURING REFERENCE LOCATION)

POINTS: 2- 6 AND 25 -27..0.001" (0.0254 mm) DIA

FOR BALANCING ON BALANCING MACHINES

POINTS: 9-22 (ROTOR BODY ON SERVICE ROTORS)........................ UP TO 0.007"(0.178 mm)
REQUIRE "RIGID" MODE BALANCING IN THREE PLANESBALANCING BY" QHSBM "©
POINTS: 9-22 (ROTOR BODY ON SERVICE ROTORS)........................ABOVE ~ 0.007" (0.178 mm)
REQUIRE ROTOR STRAIGHTENING

NOTE:
"QHSBM"© (QUASI-HIGH SPEED BALANCING METHOD" BALANCING IS DONE
BELOW MACHINE 1ST CRITICAL SPEED

Figure 14. Runout measurements for evaluation - Example

D.2. Elements of Rotor Evaluation Prior to Balancing

1. TIR data of Rotor Segments between Journals
2. Axis of Rotation (Torque input center)
3. Journal Roundness, Taper and Lobes Ovality
4. Evaluated Eccentricities at1x rev
5. Two Poles Second Harmonic
6. Journal Surface Quality surface.

D.3. Installation for Balancing on Balancing Machine

For balancing purposes on balancing machines, the bearings and supports do not have to be "the same" as those on the operating machine [21].
If a rotor has an overhung mass that would normally be supported when installed on site, a steady bearing may be used to limit its deflection and possible damage during balancing on balancing machines [15].

Non-contact transducers to measure shaft motion should be positioned inboard of bearings. The system shall be capable of measuring the once-per-revolution component of the signal. The measurement can be expressed either as amplitude and phase angle or in terms of orthogonal components relative to some fixed angular reference on the rotor.

Two vibration transducers may be installed orthogonally 90° apart at the same transverse plane to permit resolution of the transverse vibrations, pseudo-static shaft centerline movement and dynamic orbital motion, when such resolution is required.
In some cases when practical and necessary it can be beneficial to the diagnostician to install non-contact transducers outboard of the bearings, for observation of journal axial slopes relative to bearings.
In all cases, there must be no resonances or looseness of the transducers and mountings, which significantly influence relative vibration measurements referenced to Newtonian global coordinates, within the speed range of the balancing process.
The output from all transducers should be read on equipment that can differentiate between the synchronous component caused by unbalance, the slow-speed runout when significant, and other components of the vibration

Seismic transducers for measuring bearing or support vibration or bearing forces should be placed on the machine in vertical and horizontal directions for observing the absolute vibrations of stationary machine components.
The drive for the rotor should be "moment free", such as to impose negligible restraint on the vibration of the rotor and introduce negligible unbalance into the system. Alternatively, if known unbalance is introduced by the drive system, then it should be compensated for in the vibration evaluation.
To confirm that the drive coupling introduces negligible balance error, the coupling should be index balanced as described in ISO 21940-14.

E. General Description of QHSBM (Quasi-High Speed Balancing)

Balancing method of "continuous rotors" is an activity of compensating the "unbalance" effect of all eccentric masses, axially distributed randomly along rotor body RMS value of deflected rotor. In order to accomplish that, it is required placing. simultaneously, correction masses in line intersecting three balancing planes proportionally distributed axially between constraints. By doing that when rotor is accelerated we generate CF forces form **dynamic** mass axis between supports constraints (proportional to rotor angular velocity). with properly calibrated correction masses, we create equal and opposite sum of CFs to CF of pseudo static CF reduced to three points on line of centroidal mass axis. With an achieved symmetry of sums of pseudo static and dynamic RMS values of inherent eccentric mass axis, and of dynamic mass axis, the forces and moments acting at bearings elastic supports vanish. and rotor will be statically stable and dynamically balanced at any speed.

The process of reducing rotor and system vibrations is not a single isolated activity with rotor accelerated and "balanced" on a balancing machine or in operation, but the additional pre-requisite activities are required, including verification of rotor static runouts, balancing on a balancing machine in shop, ensuring rotor coupling dimensional tolerances, and bearing alignment and rotors alignment in field, to assure smooth operating machine after a major outage [22].

According to the new QHSBM balancing method, weights are distributed axially in 2N+1 (three planes) for a solution of the system rotor responses at 1st critical speed. Correction masses are distributed axially to create radial forces counteracting the axially distributed "unbalances" of the centroidal mass axis, with the objective of obtaining a sum of forces and sum of moments internal to the rotor to vanish. Reducing vibration responses at the 1st critical speed could have been achieved (according to Bishop in debate with Kellenberger), with balancing in a single plane in the middle of the rotor as well, if the intent is to resolve just the system 1st critical dynamic response falsely **assumed as rotor's body oscillating harmonic linear modal response** [8]. That assumption ignores the fact that residual unbalances and axial moments of unknown axial distribution, and responses at speeds above the system fundamental resonance speed, are referenced to different (inertial (mass axis), self-centered in space, as non-rotating reference frame. The problem with balancing large turbine and generator rotors with "significant" mass axis eccentricity or bowed rotors, using industry standard balancing methods, is in not recognizing the switch from gravity and bearing hydrodynamic established pseudo static equilibrium (SCL), and pseudo static equilibrium (SCL) controlled by inertia forces relative to self-centering centroidal mass axis [5, 10, 14, 17, 23]. Therefore the “unbalance vectors” above system 1st critical speed are referenced to different frames before and after the 1st system critical speed.
The reason Bishop was questioning whether there is a need for additional planes (more than one) in the Schenck N+2 balancing method and in his argument with Kellenberger, was because he was solving the effects of eccentricities as a rotor modal harmonic response, assuming that the excitation force vectors’ origin from unbalance(s) are at the rotating axis, coincident with the rotor mass principal (centroidal) axis, and acting radially out at the rotor center of mass.

To obtain the solution from residual “unbalances” at higher speeds, if they are significant after the “rigid” mode solution for the system 1st critical speed, Bishop proposed balancing by influence coefficients at a particular speed. By QHSBM* balancing of the 1st critical in three planes (2N+1), the **proper axial distribution** of correction masses "in phase", **rotor will be**

balanced at all speeds. In cases of residual unbalances at operating speed because of human error in placing correction mass amount and location during the balancing process, it may be necessary for fine tuning, to obtain influence coefficients from modally distributed trial weights. if predominant residual unbalance is "in phase" a "V" modal weights distribution should be utilized. If residual unbalance is predominantly "out of phase", after fully resolving "static "in phase response, an "S" modal weights distribution should be utilized.

These activities resolve all unbalances for the lateral rigid mode as well as for the axially asymmetric "rocking rigid mode", and dynamic reaction forces and bearings, and displacements of journals at the bearings will vanish. When those are resolved, that means that the rotor is brought into state of being **dynamically straight**. The rotor's principal mass axis and geometric journal centerline axis are brought to be practically coincident within values stated in ISO 1940-1 for a particular rotor and at a particular velocity of the 1st system critical speed [24]. This balancing method assures that no vibrations will be generated due to applied torque at any higher speed [16].

F. QHSBM Balancing Process Description

Prior to commencing the balancing process, it is important to verify if there is any misalignment from the drive gear, or if there is any rigid bow existing from gravity sag. That can be verified during rotor slow roll. On rotors with a static inherent mass axis deviating by a value higher than stated in ISO1940-1, for a particular machine **referenced to the system 1st critical speed [24**] system symmetry can be restored on a balancing machine by a specific procedure of compensating internal inherent mass axis axial asymmetry with external dynamic forces axially distributed in three planes (2N+1). These correction masses must be placed along the rotor body, proportionally distributed in three preselected planes between its gravity supports.

Prior to the balancing process, the surfaces under measurement transducers should be burnished to tolerance less than 0.01275 mm (0.0005"). The rotor should be run at some convenient low speed to remove any temporary bend. If transducers indicate larger values, the shaft-measuring transducer signals should be viewed in the time domain, and any "spikes" in the wave signal subtracted vectorially from any subsequent shaft measurements at the balancing speeds. **Any true runout from 1x rev eccentricity must not be subtracted from vibration signals!**

Run the rotor to some safe speed approaching the first system resonance speed. This is termed the "1st critical speed".

Balancing on high speed balancing machines by QHSBM in 2N+1 balancing planes is based on obtaining displacement readings from displacement sensors at each journal at the same angular location. The rotor should be accelerated to the maximum achievable speed up to the 1st critical speed, but to a speed corresponding to less than 90 degrees of phase angle of the critical speed peak amplitude. The reason for this is that at the critical velocity (of continuous rotors with "significant" mass axis eccentricity), corresponding to peak displacement amplitude, when above ~90 degrees, the non-rotating reference frames switch from gravity established "pseudo static" stability to an inertial (mass axis) self centering of the "pseudo static" stability axis, and the center of whirling orbit is no longer referenced to the journal centerline axis that balancing should be performed about (and that symmetry must be restored to).

Therefore, balancing is intended to be done at speeds just **below rotor critical velocity**, when the observed displacement is resulting from observed precessing of rotor rigid mode mass axis, referenced to global reference coordinates. This mass axis is defined by a straight line intersecting three points in "space", and it is precessing around the journals' pseudo-static axis in "space" (shaft centerline), up to the characteristic "critical" velocity, which is defined by the maximum achievable displacements on a polar plot from each sensor at each bearing as vectors, defined by the magnitude and direction. Two vectors, one from each journal, which represent together 100% of "unbalance" response, are trans-positioned to the rotor mid plane, lying on the mass axis line, creating a third reference point. This enables us to create a 100% response with 50% of the total in mid plane, and other 50% proportionally distributed at outboard planes proportional to respective vector size. All three correction masses should be placed "in phase". In a very unlikely special case , if phase angle difference between outboard vectors, at 1st critical speed , balancing should be performed independently for vectors in Cartesian coordinates. after obtaining rotor sensitivity to unbalance at 1st critical speed, a developed algorithm (QHSBM), converts displacement vectors to force vectors, and distribute them axially proportional to distributed "unbalances" on the rotor between rotor constraints (bearings).

G. QHSBM Balancing Process Step by Step

STEP1 Record polar plots from two sensors, select vectors at speed corresponding to maximum amplitude but at less than 90 degrees phase.

STEP2 Place trial weights at COM and obtain initial rotor sensitivity and Influence Coefficient vectors.

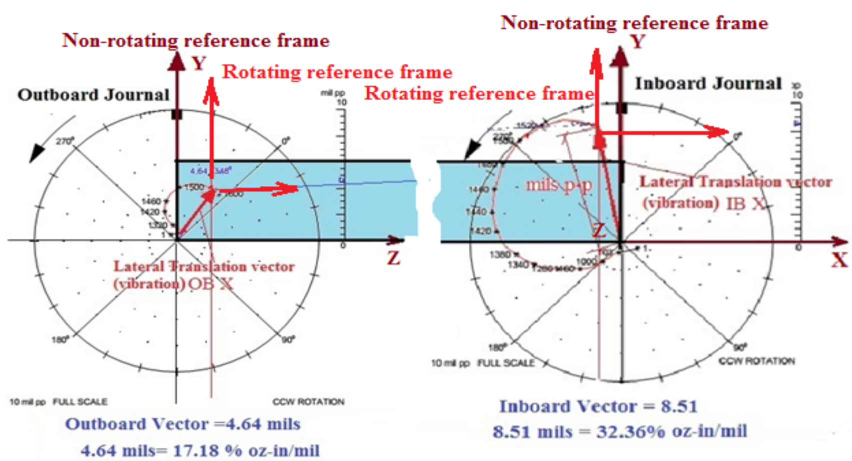


Figure 15. Polar or Cartesian vector presentation can be used as response reference

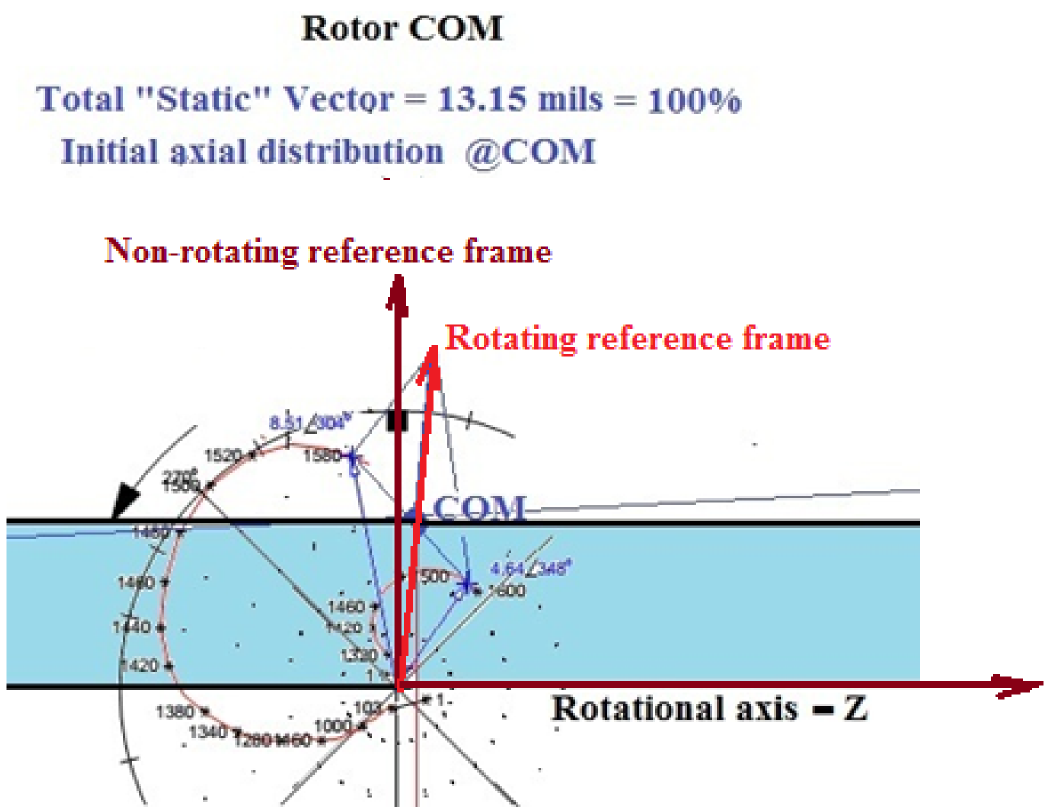


Figure 16. **Example of transferring end displacement vectors to COM and axial response vectors distribution**

STEP3 Enter initial data and IC data on QHSBM form

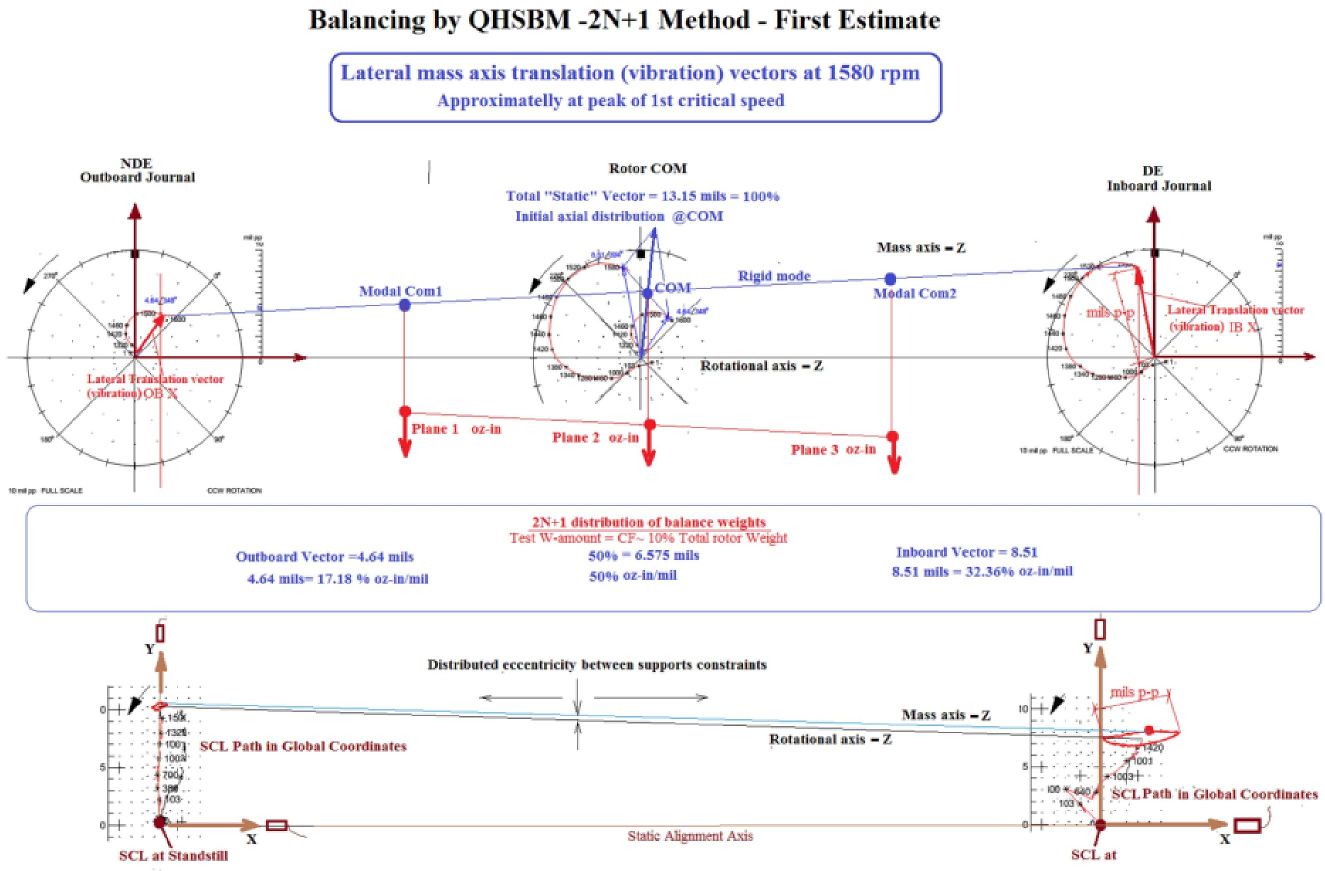
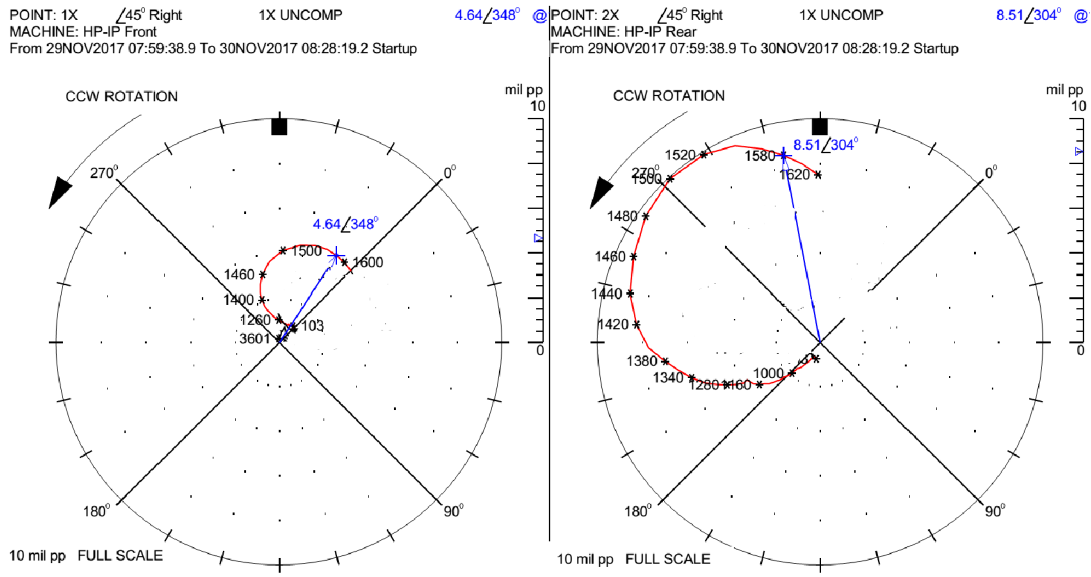


Figure 17. Example of graphical presentation of SCL, Orbits and Centroidal mass axis 3 vectors

Enter measured vectors at same speed into QHSBM data entry form. Place an estimated trial weight on the rotor at mid plane, and run the rotor to the same speed to obtain rotor "sensitivity" for conversion of displacement readings to force vectors. Enter trial weight response vector value into QHSBM form.



BALANCING PROGRAM QHSB-2N+1

PROJECT:	ARKANSAS
ROTOR:	HP-IP
DATE:	MAY 18,2018
BY:	Z.RACIC

RUN NO:	1
TIME:	0:00

WEIGHTS PLACEMENT(P) / REMOVAL(R):		
	AMOUNT / UNITS	ANGLE
L	0	0
M	0	0
R	0	0

Change only blue

	radius	axial pos		amt	angle
Left brg Vib		0		4.64	348
L Bal. plar	15.5	40.5		4.8	331.2
Mid plane	22.75	152.5		6.8	298.1
R Bal. plar	18	237		9.1	285.6
Right Brg Vib		263		9.88	283
Trial wt added to mid plane mass-radius ur				20	180
Left brg vib /w trial				1.78	336
Right brg vib /w trial				4.21	308
Mid plane (O+T vector)				3.97	303.4
			wt phase change		180
e.g: 30 oz-in change 5 mils= 6oz-in/mil			wt inf Coe	3	
		25%	L bal wts-	1.2	118.1
			mass	0.1	
		50%	Mid bal w	3.4	118.1
			mass	0.1	
		25%	R bal wts	2.3	105.6
			mass	0.126481	

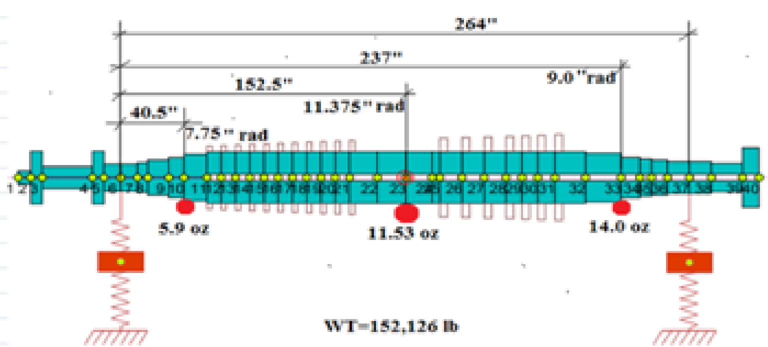


Figure 18. Obtaining "vibration displacement data and entering in QHSBM program form

STEP4 Install total amount of trial masses distributed axially in three planes according to calculated proportions by QHSBM program

If residual unbalance at operating speed requires refinement, it should be done at operating speed utilizing modal weight distribution. If predominant residual unbalance response vectors are "in phase", than a "V" modal weight configuration should be installed as trial weights to obtain ICs for further correction if necessary. If predominant residual unbalance response vectors are "out of phase", then "S" modal weights should be installed on the side on which the 1st critical response and 2nd critical response are approximately the same phase angle as observed on Polar plots. These modal distributions are essential so as not to change the net "100%" weight correction used to correct the first system critical response, but to act effectively as an axial redistribution of that determined correction.

STEP 5. If trim balancing is desired at operating speed:

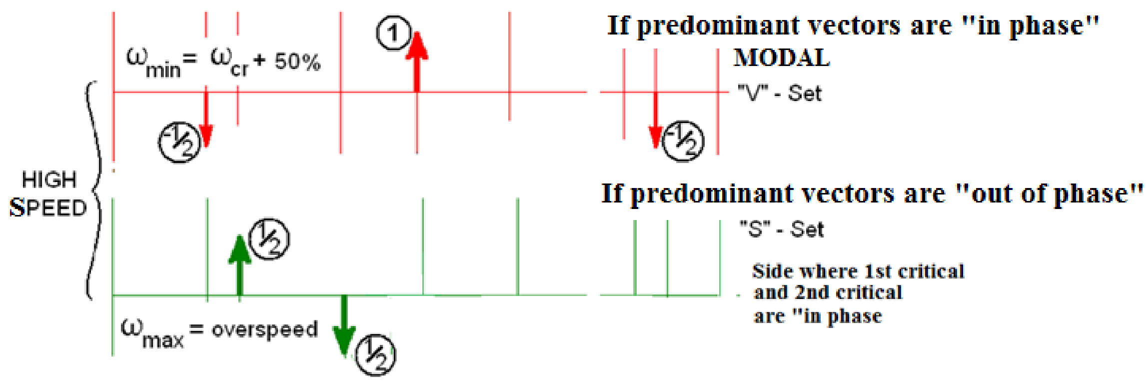


Figure 19. Modal weights placement for vibration "trimming " at rotor's operating speed

Definitions

1. **TIR (Total Indicated Runout)** is a measured deviation of a rotor's circumferentially measured segment from the referenced rotational axis.
   TIR= 2 x eccentricity + 2nd Harmonic + any other out of roundness

2. **Axis of Rotation**- a reference point for alignment purposes to another rotor rotation axis. It is also a reference to eccentricity of mass centers of rotor segments.

3. **Journal Roundness** is a measured deviation from a perfect circle, and the absence of "lobes".

4. **Evaluated Eccentricity is a vector and phase angle,** the end point of which designates the offset of the mass center of the referenced segment from the rotational axis.

5. **Second harmonic** exists on two pole generator rotors with different stiffnesses in orthogonal axial planes

6. **Journal Surface Quality** designates roughness and scrapes of the journal

G. SUMMARY

Whether a rotating body, or rotor will "vibrate" in operation or not, it depends on the difference in type of the observed "rotating" motion, based on Newton 1st law, and whether, and by how much, a constrained rotor mass axis of horizontally oriented rotor deviates radially from rotor's geometric axis in rotor rigid mode in state of rest, or in simple words, how big are runouts and bows relative to journals rotational (spin) axis.

The responsibility for assuring that rotor journals and couplings, as well as rotor body axially distributed runouts comply to design tolerances, lie in quality assurance control in machining service shops.

If rotor's body runouts, and evaluated eccentricities exceed the values from ISO1940-1 for rigid bodies at system 1st critical speed, the effects of radial centrifugal forces from randomly axially distributed eccentric masses, eccentricities can be brought to symmetry on balancing machines, by use of a new balancing method QHSBM (Quasi-High Speed Balancing Method), by simultaneously placing of correction masses, in phase, in 2N+1 balancing planes, based on responses at subcritical velocities.

Balancing by this method can be accomplished successfully if rotor runouts are measured and mathematically evaluated relative to rotor's journals centers, to determine whether correction machining at couplings and journals is necessary prior to balancing, or if QHSBM balancing program should be applied for rotor body exceeding allowable eccentricities. Runouts evaluation are possible using an appropriately dedicated FFT computer program as one developed by Z-R Consulting.

So the key to solve vibration problems of operating large turbine and generator rotors in power plants is in identifying, measuring, evaluating and correcting rotor runouts at couplings and journals, and **balancing 1st and 2nd rigid modes simultaneously in three balancing planes, at subcritical velocities**, **while rotors are in the service shop on balancing machine**, rather than waiting " with fingers crossed", until rotors are assembled in the field, and then attempting trying to salvage the situation with unit in operation, reverting in to finding an economic "tradeoff" of vibration magnitudes between that at 1st critical speed, second critical and at operating speed. With an acceptable "tradeoff", we are compromising machine dynamic condition in long term with cumulative cyclic bending moments, sometimes causing catastrophic machine failure, and typically damaging bearings and support systems in long term, from correction masses used in the current "field balancing" process methods.

It is important also to emphasize that with "field balancing" using "modal" method approach we do not eliminate the root cause of vibration, which are exclusively in the rotor body mass eccentricities, either inherent to individual rotor, or induced by misalignment of rotors' mass axes during multi-rotor turbo-generator assembly process and alignment, while in state of rest. By "field balancing" the vibrational energy in rotor is not reduced by lowering the observed responses at points of measurements, but it is transformed to "invisible" internal cyclic bending moments within rotors or couplings, causing stress, friction wear, or exciting flexible elements attached to rotor, e. g. last stage low pressure turbine free standing blades, or causing cracks ,or electrical components wear, and electrical shorts in generators.

To recognize, and to generally accept as the practical balancing method in service industry serving power generation turbine and generator rotors, the new proposed empirical balancing method QHSBM (Quasi- High Speed Balancing Method) at subcritical speeds, in 2N+1 balancing planes , whether of rigid or flexible rotors, as continuous solid body, with the eccentric centroidal mass axis relative to journals centerline rotational axis, and without the need for balancing methods based on the compromise of an economic "tradeoff", it requires verification by experimental balancing on high speed balancing machine at appropriate institution under controlled environment, in lieu of a very complex and unwieldy theoretical mathematical validation approach.

Conclusion

In this paper turbine rotor is considered a continuous solid body mass, and as such a **natural resonance** with harmonic frequencies response is a linear response of the sum of geometric rotor's finite dimensions internal molecular structure, strongly associated with rotor material physical properties, with rotor in a free-free state, when it is excited through a direct interaction by an external dynamic force vector, perpendicular to rotor's centroidal mass axis. Rotor natural resonance frequency response is a function of square root of rotor rigidity(tensile strength) relative to rotor geometric body mass length "L" between internal constraints (nodal points).

Rotating machine system flexible rotor with eccentric mass axis, constrained by gravity in hydrodynamic bearings, exhibits amplified response when it is accelerated through system critical velocity frequency range. From the above facts we can conclude:

1. **W.J Macquorn Rankine** as the first who performed an analysis of a spinning shaft (1869), **was wrong** , when he had predicted that the shaft bends considerably and whirls in that bent form. That speed he named a "Whirling speed" of the shaft. He concluded that beyond this whirling speed the radial deflection on his model increases asymptotically, and his conclusion was that no shaft can operate beyond that speed.

2. **DeLaval had proved that Rankine was wrong** with his experiment, operating flexible rotor way above "1st critical speed". **DeLaval was correct**, but without ever finding that the reason for that is the switching of relative reference frames in quantum field. Scientist later atriuted the event to " external and internal damping", creating a mathematical proof

3. Later researchers, based on certain assumptions and boundary conditions and assumming rotor as a closed system developed equation of motion of rotating bodies based on Newton laws and equating rotating motion with linear motion and with continuous rotor assumed as a point mass, based on the hypothesis that 1st critical speed is a non linear amplification of motion from self-excitation of rotor assummed as a point mass, caused by a fictitious force in external spring damper system.

4. As rotating machines operating speed and operating efficiency was pushed higher and higher, the issue of **rotor stability** became the primary issue in industry. When external damping was proven insufficient in explaining rotors instabilities at operating speed and maximum designed load in some cases, additional hypothesis were developed like rotor internal "damping" and material hysteresis*, but still **assuming that the "1st critical speed is linear oscillating motion of rotor fundamental harmonic response.**

* (*the lag in response exhibited by a body in reacting to changes in the forces, especially magnetic forces, affecting it. The phenomenon exhibited by a system, often a ferromagnetic or imperfectly elastic material, in which the reaction of the system to changes is dependent upon its past reactions to change*).

5. The study shows that a continuous flexible rotor with eccentric mass axis actually undergoes a rigid (pseudo static) deformation, when accelerated through the critical velocity frequency range destabilizing dynamic equilibrium of subatomic particles in quantum field.

6. Rotor **stability threshold speed** and dynamic equilibrium is established between the apparent gravity force on bearing and oil friction at minimum oil film thickness. This stable state is of short duration, and exists at super-critical velocity (end of critical speed velocity range and the state of least action) and phase angle of 180 degrees, between excitation force vector of torque moments (at COM of mass axis as rigid constraint in space) as non rotating inertial reference frame, and journals axis as rotating frame whirling non-inertial reference frame.

7. As rotor is accelerated above supercritical velocity frequency, mass axis (non-rotating reference frame is a SCL- precession up to 90 degrees and gravity stability axis at 180 degrees. Dynamic orbit centerline (non-rotating frame - inertial stability axis through supercritical angular velocities and machine load increase at constant velocity) is **continuously self-centering*** in space, forced by inertia forces from the residual axially distributed mass eccentricities. Journals centers during mass axis self-centering at same time are whirling as gyroscopic rotating pendulums, pinned at COM and flexibly constrained at respective bearings and supports,

proportional to torque moments at particular rotor angular velocity.
** Self- centering of mass axis (non-rotating, inertial reference frame) is a pseudo static path-motion in space and time, can be graphically presented in SCL plot referenced to Newtonian absolute reference frame. When the path hysteresis between rotor acceleration by external torque, and deceleration by rotor inertia, during rotor balancing on high speed balancing machine is greater than 0.001", it indicates the existence of "residual unbalances" from a "significant" rotor body mass axis eccentricity, relative to journals centerline axis with rotor at rest. This condition can be verified in advance with rotor "runouts" evaluation as 1xrev eccentricities and 2xrev ovality of rotor body , by FFT analysis.*

8. Study has shown that rotor fundamental harmonic **resonance frequency** of flexible rotor in free state in gravity environment **is not equivalent to 1st critical speed** of the system's with flexible supports **forced resonance frequency response!**

9. Accordingly, balancing methods of large turbine and generator rotors should be balanced, not based on axially distributed point mass "unbalance vectors", but based on compensating the inherent longitudinal mass axis, between bearings constraints with rotor in state of rest, by dynamic forces of compensating masses axially distributed in three planes, forming a dynamic mass axis mirror imaging the internal, inherent, mass axis (line intersecting centers of masses of rotor Null mode eigenvector of harmonic *).

**The objective of balancing rotors is to vanish bearings dynamic reaction forces, and reduce journals radius of orbital precessional motion at operating speed to magnitude of internal , inherent mass axis eccentricity, visible and measurable as displacement, but with generating no forces on bearings and supports*

Postscript

Balancing of large turbine and generator rotors is a mechanical technological process evolved from physics theory of rotordynamics of closed systems based on boundary conditions with many assumptions, which in real life, specifically in segment of balancing large turbine and generator rotors, experiments are often in contradiction to theoretical prediction. This was nicely summarized in following paragraph, paraphrased from **"Principles of Physics by Serway; V. Gordon Lind, Utah State University":**

"Physics is a study of the laws and principles of all phenomena in nature. One of branches of physics is Classical Mechanics, the study of matter in motion and its causes, i.e. forces. Students in Classical mechanics all over the world have learned to trust the physics theories. Discrepancies between theories and experiments are now unexpected. Physics and science in general have to be based on theories, laws and principles that agree with experimental observations and these do. Experimental results in disagreements with established theory are probably wrong. But if discordant results are repeatable and verified, then theory in such event, no matter how appealing, popular and generally accepted (op. ed. Balancing Theories), must be corrected and brought into agreement with experiment".

H. Bibliography and references

1. Benjamin S. &Blanchard | Pearson :" Systems Engineering and Analysis" Prentice Hall International Series in Industrial & Systems Engineering;5th Edition, 2010

2. Rubens M. Marinho Jr.:" Noether's theorem in classical mechanics revisited Departamento de FisicaInstituto Tecnologico de Aeronautica, Brazil

3. Peter D. Hylton**:** "Low Speed Balancing for Supercritical Shafting in gas Turbines"; Proceedings of ASME Turbo Expo 2008: Power for Land, Sea and Air ;GT2008, June 9-13, 2008, Berlin, Germany;GT2008-50077

4. F.F. Ehrich: "Pseudo-High- Speed Balancing"; Transactions of the ASME, 418/ Vol 112, October 1990

5. "Dynamics of Timoshenko beam on an elastic foundation" by Archadiy Manewich; Proceedings of the 3rd International Conference on Nonlinear Dynamics-ND-KhPI 2010; September 21-24, 2010, Kharkov, Ukraine

6. "The Timoshenko Beam on an Elastic foundation and Subject to a moving step Load", Part 1: Steady State Response

7. Part 2: Transient Responseby S.F. Felszeghy :College of Engineering, Computer Science, and Technology Department of Mechanical Engineering Professor Emeritus of Mechanical Engineering

8. Ferrari via Vignolese 905, Modena, Italy (e-mail: andrea nobili@unimore.it

9 "General analysis of Timoshenko Beams on Elastic Foundation"by S.Abohadima, M Taha and M.A.M Abdeen; Hindawi Publishing Company; Mathematical problems in Engineering, Volume 2015

10 Stability of a Timoshenko Beam Resting on a Winkler Elastic Foundation by S.Y Lee, Y.h. Kuo and F.Y. Lin; Journal of Sound and Vibration (1992) 153(2), 193-202

11. "Superposed Standing Waves in Timoshenko Beam" by K.T. Chan, X.Q. Wang, R.M.C. So and S.R . Reid; 10.1098/rspa.2001.0855- The Royal Society

12. Giordano, J. and Zorzi, E. (1985): "HPOTP Low-Speed Flexible Rotor Balancing", Prepared for NASA Marhall Space Flight Center, by MTI, doc # 85TR28

13. Racic Z, Racic M. (2014): "Development of a New Balancing Approach for Significantly Eccentric or Bowed Rotors." In: Mechan. Machine Science, Vol. 21, Paolo Pennacchi(Ed); Proceedings of the 9th IFToMM International Conference on Rotor Dynamics, Milan, Italy, 22-25 Sept 2014.

14. Racic Z, Racic M. (2014): "Behavior of Eccentric Rotors through the Critical Speed Range." In: Mechan. Machine Science, Vol. 21, Paolo Pennacchi(Ed): Proceedings of the 9th IFToMM International Conference on Rotor Dynamics, Milan, Italy, 22-25 Sept 2014.

15. . Hidalgo J, Dhingra A. " High Speed Balance of Rotors With Overhangs: When is Overhang Likely to Cause Problems?" Journal of Testing and Evaluation, May 2006, Vol.34, No3.

16 . Z.Racic, J.Hidalgo "PRACTICAL BALANCING OF FLEXIBLE ROTORS FOR POWER GENERATION", Proceedings of the ASME 2007 IDETC/CIE 2007 September 4-7, 2007, Las Vegas, Nevada, USA

17 Z.Racic, M. Racic, A.Zyvotov- "Updated Theory of Rotor Dynamics"; For presentation at Dyrobes Seminar, October2012

18 Abeyaratne, Rohan, 1952 –Lecture Notes on The Mechanics of elastic solids. Volume II: Continuum Mechanics/ Rohan Abeyaratne – 1st edition – Cambridge, MA and Singapore: ISBN-13:978-09791865-0-9

19 Physics of music- Resonance ; http://kellerphysics.com/acoustics/Lapp.pdf

20 Kliemann, W., Langford, W. F., & Namachchivaya, N. S. (1996). Nonlinear dynamics and stochastic mechanics. Providence, R.I: American Mathematical Society. Crandall, Ch

21 https://z-rconsulting.com/pdf/Analysis%20and%20Solution%20of%20 Abnormal%20Vibration%20Problem%20of%20Exciter.pdf 22 Racic, Z. and M. Racic: "Proactive shop strategy for a successful turbine generator rotor outage". International Journal of COMADEM, vol. 19 no. 4, (2016).

23 https://z-rconsulting.com/pdf/Racic%20%20-%20IFTOMM%202018%20-%20final%20paper.pdf

24 Z-Racic- (Personal archive-unpublished); Expanded interpretation of ISO 1940-1 and the Effect on Balancing and Stability of Continuous Flexible Turbine and Generator Rotors with Eccentric Mass Axis

25. Von Bertanllaffy, L.: *General Systems Theory*; George Braziller Incorporated, (2015)

26. Neuenschwander, D. E. (2011) Emmy Noether's Wonderful Theorem, The John Hopkins Univ. Press.

27. Erik Swanson, et al, Xdot-Consulting, Chapel Hill North Carolina; Sound and Vibration 2005

28 Rolling Motion : **University of Central Florida Pressbooks;** A Resource for Academic & Educational Publishing at UCF (https://pressbooks.online.ucf.edu/)

29. . Dr David Tong: " Quantum Field Theory"; University of Cambridge Part III Mathematical Tripos

30. NASA Conference publication 3026: Rotordynamic Instability Problems in High-Performance Turbomachinery